An agent-based model of urban housing markets: may the socio-spatial segregation preserve some social mix?

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Introduction

The place where people live and the way they distribute across the cities matter.

This article:

$\Rightarrow$ **seeks to explain how individuals with different willingness to pay are distributed over a monocentric city.**

$\Rightarrow$ **deals with the dynamic of prices formation in this city.**

The post-Schelling literature

- Measure the segregation (level of segregation and social interactions) . *Echenique and Fryer 2007*, QJE,
- Correlate measures of segregation (ethnic, religious and linguistic) with measures of quality of policymaking. *Alesina and Zhuravskayay*, *2011*, AER ; *2024*, SSRN
- Evaluate the effect of segregation on the socioeconomic performance of minorities (*Jenks and Meyer 1990* or *Cutler et al. 2008*, *Journal of Urban Economics*).
- Segregation associated with conflict (*Corvalan and Vargas (2015)*, JDE

⇒ **The segregation we consider in this article is purely related to income distributions**

The formation of prices on the real estate market

Rosen (1974), **hedonic prices**

- The price of a square meter depends both on intrinsic variables and on extrinsic variables concerning the area and the facilities.
- *Brueckner et al. (1999)* : the relative location of different income social groups depends on the spatial pattern of amenities in a city.
- The role of the quality and density of the neighborhood, the reputation of the neighboring schools and the level of security matter: *Ioannides (2003)*, *Figlio and Lucas (2004)*, *Bono et al. (2007)* or *Seo and Simons (2009)*.

⇒ **In our model, the formation of prices depends both on an intrinsic attractiveness and a dynamic subjective one which depends on the willingness to pay of the agents.**

Whom do people prefer to live with?

∗ A part of the happiness literature tends to prove that people are more happy when they have poorer neighbors.
∗ Ferrer-i-Carbonell and Ramos (2021). Strong correlation between inequality and happiness.

- *Goyal and Ghiglino (2010)* poor people lose, while well endowed individuals gain, as they move from a society which is economically segregated to an integrated society. (*contradicts other articles*)
- *Luttmer (2005)*: utility functions depend also on relative consumption
- *Clark and Oswald (2002)*: following a cohort of unemployed people.

⇒ **In our model, we postulate that people prefer to live with people who are as rich or richer than they are.**

Overview

A model of residential location:

- People **take their decision** according both to their willingness to pay (WTP) and their individual evaluation of the level of attractiveness of the different locations.
- Buyers, **heterogeneous by their WTP** are looking for flats, according to the level of attractiveness of the place where the flat is located.
- Agents are **both buyers and sellers**
- **The intrinsic attractiveness** A^0 is simply defined as depending on the distance to the geographical center. Max at the center and decreases with the distance, (Alonso and von Thünen) . Form of A^0 easy to generalize in order to obtain a polycentric city.

Plan

- Theoretical model: assumptions and main analytical results
- Simulation results and extend
- A comparison with Paris transactions on the real estate market.

Theoretical model: the assumptions

- **A0: Space** A finite number N of homogeneous goods are located on a discrete set Ω of locations X uniformly distributed in a bounded bi-dimensional space.

- **A1: Agents** Time is discrete and indexed by t. The time increment is δt. The horizon is infinite.
  At each period, a finite number of agents, in one of the three following states: (1) **buyer, (2) seller, (3) housed.** Housed agents become sellers at a homogeneous rate α. At t a constant number of new buyers arrive on the market.
  They add to the buyers who did not succeed in the previous period.

A2: Demand Prices

- Agents are characterized by their willingness to pay (or sell). $\rightarrow$ the maximum price the agent is ready to pay for an asset.
  Let's consider a finite number K of WTP. Agents with the same willingness are designed by k-agents, $k \in \{0, ..., K-1\}$, and have WTP P_k, $P_0 < P_1 < ... < P_{K-1}$.
  When the agent is acting as a buyer, his demand price is $P_k^d = P_k$.

A3: Attractiveness

The total X-attractiveness at time t seen by a k-agent is updated at each step of the dynamics according to:

$$\Delta A_k = \underbrace{\omega \Delta t (A^0(X) - A_k(X,t))}_{\text{Relaxation term: when no transactions} \to 0} + \underbrace{\epsilon v_{k_>}(X,t)}_{\text{New buyers with WTP} \geq P_k}$$

When ω is large, the attractiveness reacts very quickly and tends to zero as soon as there is no transaction.
When ϵ is large, the decision of buying is strongly influenced by the neighborhood.

The average X-attractiveness :

$$\bar{A}(X,t) \equiv \sum_{k=0}^{K-1} \frac{A_k(X,t)}{K} \quad (1)$$

A4: Offer Prices

Each seller $\Rightarrow$ **an offer price which depends on his willingness to sell and on the mean attractiveness of the location.**
A k-agent, when acting as a seller, has his willingness to sell determined by his WTP P_k.
The resulting offer price $P_k^o(X)$:

$$P_k^o(X) = P^0 + \underbrace{(1 - \exp(-\lambda \bar{A}(X,t)))P_k}_{\to 0 \text{ when } \bar{A}(X,t) \to 0.} \tag{2}$$

$P^0 \Rightarrow$ minimum price of an offer
λ is a parameter.

A5: The Matching (1)

At each time step:

- The density ρ_k of k-buyers is the sum of a constant proportion $\frac{\gamma}{K}$ of new buyers, and of the density of buyers who have not yet succeed.(outsiders) Each agent has a probability $\pi_k(X)$ to visit a given location X, that depends on the attractiveness of the location:

$$\pi_k(X) = \frac{1 - \exp(-\lambda A_k(X,t))}{\sum_{X' \in \Omega} 1 - \exp(-\lambda A_k(X',t))}. \quad (3)$$

A5: The Matching (2)

- At location X, a transaction between a k-buyer and a k'-seller with offer price $P^o_{k'}(X)$, **can be realized if $P_k > P^o_{k'}(X)$.** The transaction

  price :

  $$P_{tr} = (1 - \beta)P^o_{k'}(X) + \beta P^d_k \tag{4}$$

  where β is a constant coefficient.

Theoretical Model: the equilibrium

A Continuous time dynamics

- The total density $u_k(X,t)$ of **k-insiders** is:

$$(1-\alpha)\partial_t u_k(X,t) = v_k(X,t) - \alpha\, u_k(X,t) \tag{5}$$

At each period, a proportion α of people leaves the insider side.

- A k-agent continuously **updates the attractiveness** of a location X :

$$\partial_t A_k(X,t) = \omega\,(A^0(X) - A_k(X,t)) + \epsilon\, v_{k_>}(X,t) \tag{6}$$

- The density of **k-outsiders** $\rho_k(X,t)$ is:

$$\rho_k(X,t) = v_k(X,t) + \bar{v}_k(X,t), \tag{7}$$

Stationary state: the general case

The equilibrium: the stationary state (whenever it exists) of the dynamics, i.e. $\rho_k(X,t), A_k(X,t)$, $u_k(X,t), v_k(X,t)$, become constant.

$$
\begin{aligned}
v_k^*(X) &= \alpha u_k^*(X), \quad \textit{offer} = \textit{demand} &(8)\\
A_k^*(X) &= A^0(X) + \frac{\epsilon}{\omega} v_{k_>}^*(X) &(9)\\
\rho_k^*(X) &= \pi_k^*(X) \sum_{X' \in \Omega} \rho_k^*(X'). &(10)
\end{aligned}
$$

which gives:

$$\sum_{X \in \Omega} v_k^*(X) = \alpha \sum_{X \in \Omega} u_k^*(X) = \frac{\gamma}{K}. \tag{11}$$

$\Rightarrow$ The total density of k-transactions during a step is $\frac{\gamma}{K}$.

$\Rightarrow$ **The total number of insiders is the same for all the k values.**

Definition: non-saturated equilibrium

. *A non-saturated equilibrium is defined as an equilibrium where, for any given* $k \in \{0, ..., K-1\}$, *at any location* $X \in \Omega$, *either the WTP of the* k*-agents are too low, so that none of them can afford to buy a good at this location, hence* $v_k^*(X) = 0$, *or the* k*-agents can afford to buy a good at this location, and in such case any* k*-demand is satisfied, that is*

$$\bar{v}_k^*(X) = 0. \tag{12}$$

$\Rightarrow$ **occurs if the inward flux is not too large compared to the outward flux and to the total number N of goods at a given location.**

Above the threshold

Let us first assume $P_c^* \leq P_{K-1}$, hence $\bar{k} \leq K-1$. We consider the k-agents with $k \geq \bar{k}$: **they are rich enough to buy a flat wherever they want**. The market is non saturated:

$$\forall k \geq \bar{k} \ , \ \forall X \in \Omega, \ \bar{v}_k^*(X) = 0. \tag{13}$$

Proposition 1: The density of housed k-agents with $k \geq \bar{k}$ does not depend on the level of individual WTP but only on the intrinsic attractiveness of the location, according to:

$$\forall k \geq \bar{k} \ \ \forall X \in \Omega, \ u_k^*(X) = \frac{\gamma}{K\alpha} \frac{A^0(X)}{Z^0} \tag{14}$$

with

$$Z^0 \equiv \sum_{X \in \Omega} A_0(X). \tag{15}$$

Below the threshold

The budget constraint matters. Not everybody is rich enough to buy a flat wherever he wants.
$\Rightarrow$ minimal distance $d_c^*(k)$ from the center at which a k-agent can buy a flat? **As one would expect, we find that the lower the WTP, the higher the distance.**

Proposition 2: For $k < \bar{k}$, the density of k-insiders depends on the level of individual revenues and on the intrinsic attractiveness of the location according to:

$$u_k^*(X) = \frac{\gamma}{K\alpha} \frac{A^0(X)}{\sum_{X' \in \Omega_k} A^0(X')} \text{ if } D(X) \geq d_c^*(k) \quad (16)$$

$$= 0 \text{ otherwise} \quad (17)$$

where

$$\Omega_k \equiv \{X \in \Omega | D(X) \geq d_c(k)\} \quad (18)$$

and $D(X)$ is the distance to the center.

Consequences

$\Rightarrow$ For each k-agents, **a critical distance** can be computed:

$$d_c^*(k) = R\sqrt{-\ln\left[\frac{\frac{1}{\lambda A_{max}}\ln\frac{P_k}{P^0}}{1+\frac{\epsilon\gamma}{2\omega Z^0}[\frac{K+1}{K}-\frac{(k+1)k}{K^2}]}\right]} \quad (19)$$

where, $Z^0 = \sum_{X\in\Omega} A_0(X)$. This equation is valid whenever the argument of the logarithm is smaller or equal to 1, and otherwise $d_c^*(k) = 0$.

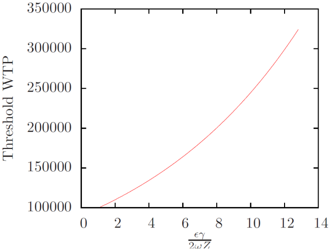
$\Rightarrow$ **The averaged transaction prices** on a site X, denoted $\langle P_{tr}(X)\rangle$, may be written as follows :

$$\langle P_{tr}(X)\rangle = (1-\beta)P^0 + [1-(1-\beta)\frac{P^0}{P_0+\Delta\frac{k_0}{K-1}}][P_0+\frac{\Delta}{2}(1+\frac{k_0}{K-1})] \quad (20)$$

The critical threshold value with respect to the dynamic component of the attractiveness

$\downarrow$ **Rich people**

$\downarrow$ **Rich people**



$\uparrow$ **Poor people**

$\uparrow$ **Poor people**

Dynamic used for numerical simulation

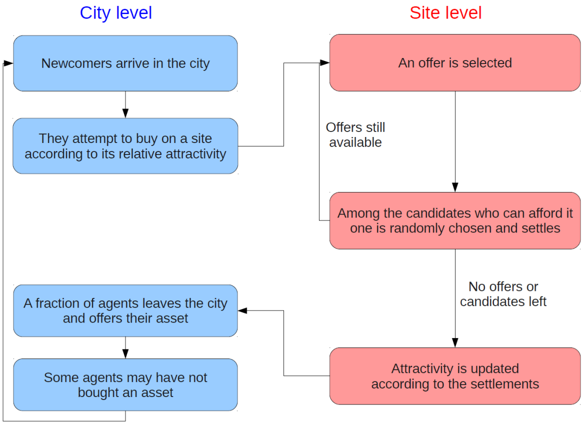


Figure: Model dynamics.

Model overview

Real estate transactions between heterogeneous agents

- Square lattice $\equiv$ city
- Each cell of the lattice is characterized by an attractiveness
- N homogeneous assets available on each cell of the lattice
- Prices of these assets depend on the attractiveness
- Agents classified by categories arrive on the lattice
- Category $\equiv$ buying ability
- These agents prospect on a site according to its relative attractiveness
- A settlement is achieved only if buying abilities $\in$ prices of the offers
- Attractiveness updated according to the last settlements
- Departure and newcomers

The values of the parameters: $\lambda = 0.1$, $A_{max} = 1$, $\epsilon = 0.022 * L^2$, $R = 10$, $\Delta = 225000$, $\omega = \frac{1}{15}$, $P^1 = 200000$, $\frac{\gamma}{K} = \frac{1000}{L^2}$, $\beta = 0.1$, $N = 200$. The initial number of offers is voluntarily high enough to prevent the system from saturating, i.e. when the number of available assets becomes 0 on some sites.

Socio-spatial segregation and income mix

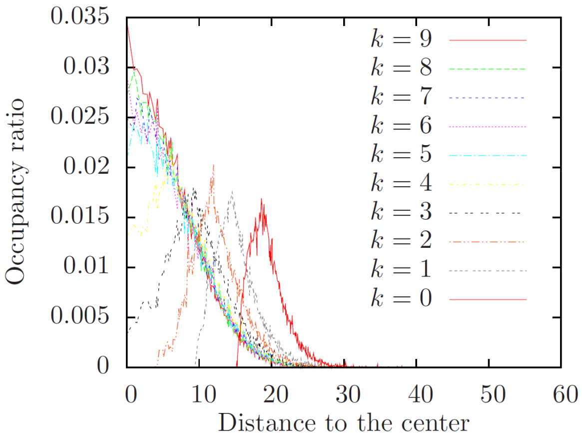


Figure: $K = 10$ revenue are present on the lattice.

Social mix index

The segregation is measured using the index proposed by Duncan and Duncan (1955) (alternative possibility: measure of entropy.)

$$ID(X) = \sum_{k=0}^{K-1} |\nu_k(X) - \frac{1}{K}|. \qquad (21)$$

where $\nu_k(X)$ is the relative density of k-agents:

$$\nu_k(X) \equiv \frac{u_k(X)}{\sum_{k=0}^{K-1} u_k(X)} \qquad (22)$$

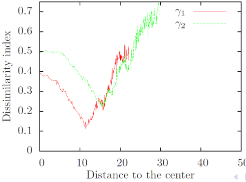


Figure: Dissimilarity index for two rates of newcomers γ_1 and γ_2

Comparison between analytical and simulation results (1)

Where do rich people live?

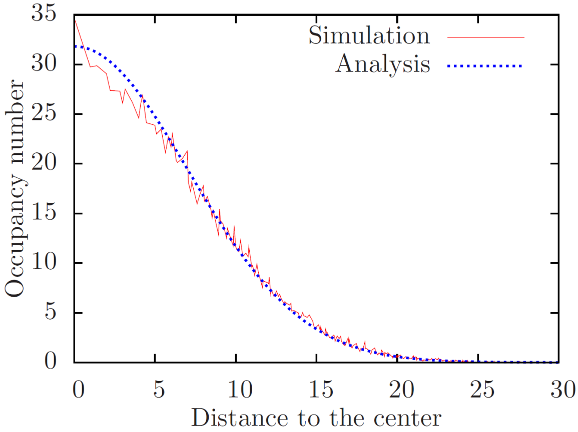


Figure: WTP $k = 9$ ($K = 10$)

Comparison between analytical and simulation results (2)

Where do poor people live?

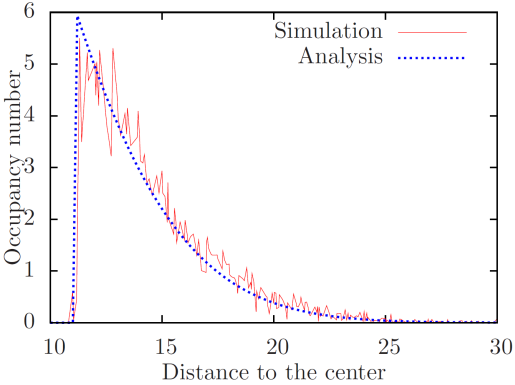


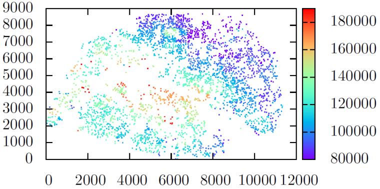
Figure: WTP $k = 2$ ($K = 10$)

Figure: WTP $k = 2$ ($K = 10$)

Comparison with empirical data

B.I.E.N. database, organized by the "Chambre des Notaires de Paris" which registers real estate transactions for Paris and Ile De France. focus on year 2003

Prices distribution over Paris



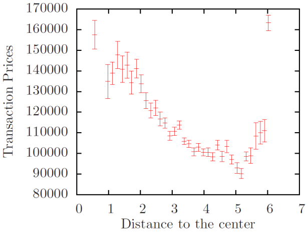


Figure: Les prix diminuent avec la distance au centre puis remontent (effet 16ème arrondissement).

Commentaire: On rate l'effet "18/19/20".

Comparison with empirical data

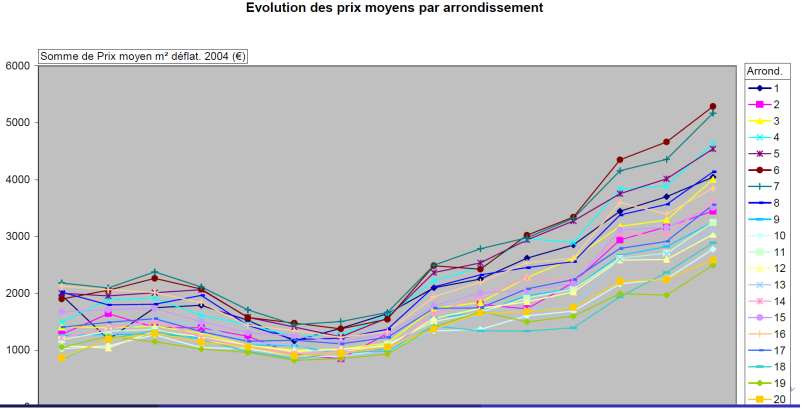


Figure: The evolution of averaged transaction prices across time and arrondissement.

Comparison with empirical data

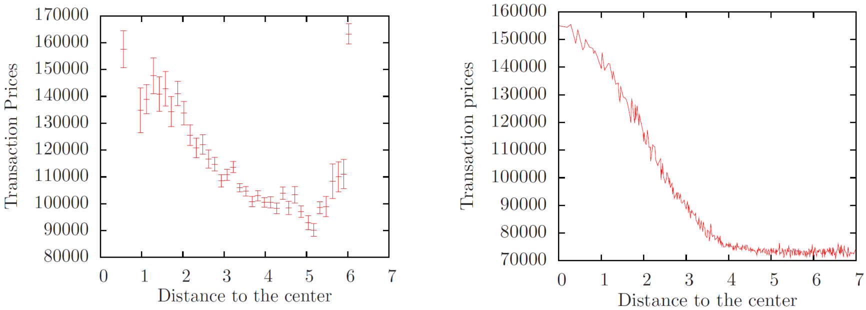


Figure: Averaged transaction prices with respect to the distance to the center (left: data, right: model).

The standard deviation of the transaction prices

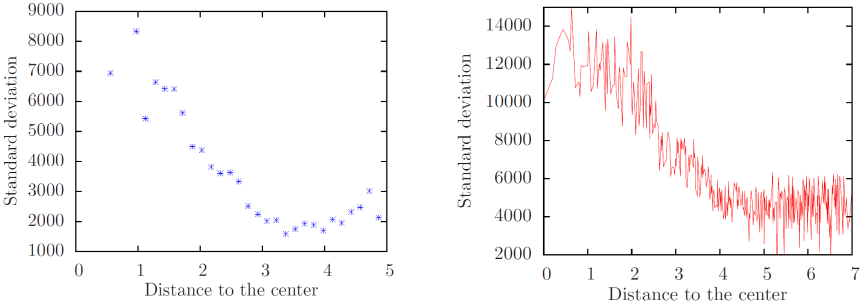


Figure: Standard deviation of the averaged transaction prices with the distance to the center (left: data, right: model).

Outcomes and Openings

$\Rightarrow$ We consider agents heterogeneous in their willingness to pay $\rightarrow$ go beyond the simple Schelling case with binary status.

$\Rightarrow$ **Three insights.**

1. A socio-spatial segregation, richer people near the center and poorer people at the periphery.
2. Existence of an area of social mix.
3. The variance at the center, where the higher prices are localized higher than at the periphery.

$\Rightarrow$ **Openings.**

1. What's about agents preferring to live in working class district?
2. Looking at the diffusion of prices.
3. Existence of other type of segregation in the rich district.