Elasto-plastic solution - Morh-Coulomb criterion

The effect of face advance can be taken into account by applying a varying fictitious internal pressure pf on the tunnel wall (Panet. 1995).

$$p_f = (1-\lambda)\sigma_0$$

In axisymmetric conditions the Mohr-Coulomb failure criterion is written as

$$F(\sigma_\theta, \sigma_r) = \sigma_\theta - K_p\sigma_r - \sigma_c = 0$$

with

$$K_p = \frac{1 + sin\phi}{1 - sin\phi}$$

$$\sigma_c = \frac{2Ccos\phi}{1 - sin\phi}$$

and the plastic flow rule

$$\beta\Delta{\varepsilon_\theta}^p + \Delta{\varepsilon_r}^p = 0$$

with

$$\beta = \frac{1 + sin\psi}{1 - sin\psi}$$

$$C - Cohesion; \;\; \phi - Friction\ angle; \;\; \psi - Dilation\ angle$$

We define a critical deconfinement rate

$$\lambda_e$$

when the plastic zone occurs on the tunnel wall. This critical value can be obtained from the failure criterion

$$F((1+\lambda_e)\sigma_0, (1-\lambda_e)\sigma_0) = 0$$

or

$$\lambda_e = \frac{1}{K_p + 1}\left(K_p - 1 + \frac{\sigma_c}{\sigma_0}\right)$$

In polar coordinates the equilibrium equation without body forces can be written as

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

Substituting the yield criterion in the equilibrium equation we can obtain

The stress field

In the plastic zone (R ≤ r ≤ Rp)

with Rp the plastic radius

$$\frac{R_p}{R} = \left(\frac{2}{K_p + 1}\frac{(K_p - 1)\sigma_0 + \sigma_c}{(1-\lambda)(K_p - 1)\sigma_0 + \sigma_c}\right)^{\frac{1}{K_p - 1}}$$

then

$$\sigma_r = \frac{\sigma_c}{K_p - 1}\left[\left(\frac{r}{R}\right)^{K_p - 1} - 1\right] + (1-\lambda)\sigma_0\left(\frac{r}{R}\right)^{K_p - 1}$$

$$\sigma_\theta = \frac{\sigma_c}{K_p - 1}\left[K_p\left(\frac{r}{R}\right)^{K_p - 1} - 1\right] + K_p(1-\lambda)\sigma_0\left(\frac{r}{R}\right)^{K_p - 1}$$

In the elastic zone (r ≥ Rp)

$$\sigma_r = \left(1 - \lambda_e\left(\frac{R_p}{r}\right)^2\right)\sigma_0$$

$$\sigma_\theta = \left(1 + \lambda_e\left(\frac{R_p}{r}\right)^2\right)\sigma_0$$