COURSE ON TRANSPORT SURVEY METHODS

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Course on Transport Survey Methods

A Data requirement and collection in transport planning

The rational approach to transport planning makes an intensive use of information on trip volumes and conditions, interpreted as measures of respectively the quantity and the quality of the transport service. Detailed, reliable information is required to enlight decision-making on the transport system.

This provides the basic motivation for the course on transport survey methods. As various methods are available to produce information, some theory is required to introduce them, to assess their respective scope and associated costs, and to analyze their potential contribution to the information system (particularly to a model of the transport system). Traffic and transport theory is relevant to describe the transport system, to characterize it on the basis of certain state variables. Statistical theory is relevant to estimate state variables based on partial observation, which is the general case. It is also useful to analyze the accuracy of estimated variables, hence to make trade-off between measurement cost and accuracy.

Therefore the course is comprised of three kinds of lessons: case studies, lectures on traffic and transport theory, statistical lectures. The lectures include exercises to get hands on the concepts. It is hoped that the three kinds of lessons are alternated in a pleasant order, easing the way to an in-depth understanding. The overall thread of the course is a gradual one, from most aggregate to most disaggregate information. This thread enables us to provide the relevant statistical concepts in a progressive manner, and it may be considered as an important pedagogic feature of the course.

The introductory lesson aims at making more precise these claims. It provides definitions for the state variables of the transport system, followed by an overview of the main data collection methods.

A1 State variables for the transport system

State variables are quantities which describe the state of a given system. They contain objective information which can enlight decision-making.

A1a The objectives and constraints of transport planning

A transport system may be decomposed into four subsystems: a demand component including consumers, a supply component comprised of suppliers, a public authority often in the form of a local government, and an impactee component which includes land-users and the general environment (ecological as well as socio-economic). The four components entertain a rich interplay, from the combination of activities located in different places, to the undesirable impacts of injuries, noise and pollution.

Let us review the objectives and constraints of each component, in order to reveal the state variables of prominent interest.

Demand. Transport consumers are passengers, shippers or carriers. Transport demand derives from the demand for activities: the basic value of transport pertains to the activities which it makes available, not to the move in itself. From the consumer viewpoint, the availability of destination places at given times is the primary function and impact of transport. Assuming availability, performance (short duration, comfort, safety) and cost efficiency are the next desirable features.

Supply. As in any economic sector, the basic objective of a supplier is to make profit. As profit is the difference between revenues and production costs, the maximization of profit may be achieved by minimizing the production costs and maximizing the revenues, which include commercial revenues from the customers and subsidies from the public authority. As transport is a highly capitalistic sector, each investment requires a long amortizement time: hence system stability and perenniality is also desired by the supplier.

Impactees. The primary impact of transport is to allow the users to benefit from several activities located in different places. However there are also side impacts that may affect the transport users or other groups of impactees, including users of other transport modes (eg. pedestrians affected by car traffic), land-users, inhabitants etc. Let us mention the following negative side impacts: (i) congestion, (ii) accidents, (iii) chemical pollution of air and soil, (iv) noise, (v) space occupancy. A counterpoint is the additional land value derived from transport accessibility, particularly for those properties close to public transport stations.

Most side impacts depend on the intensity of traffic and on the flow conditions (eg. emission of gas pollutants as a function of number of vehicles and operating speed). The associated objective is to maximize the positive impact of accessibility and to minimize the negative side impacts.

Public authority is responsible for welfare of the whole activity system, including social as well as economic activities. It decides on infrastructure planning, operating and commercial rules, vehicle standards in order to mitigate the positive and negative impacts of transport. Decision-making is prepared and enlightned by cost-benefit analysis, which evaluates a transport scenario in a global way, including not only the economic impacts but also the equivalent economic value of other impacts such as safety, noise or environmental damage. This requires to measure the impacts: most measurement methods are indirect and consist in numerical computations based on traffic measures.

A1b Dimensions of analysis and of disaggregation

Traffic extends over space and time; it concerns various economic actors at varying degrees. The knowledge of traffic impacts involves not only the basic questions of "which impacts" and "how much impact", but also the imputation questions of "which impactees", "which impacter", "at which degree". In other words, traffic analysis must often be disaggregated with respect to location, time, economic actor, type of traffic (passenger or freight, urban or interurban, low cost or high quality).

The spatial and temporal circumstances also provide dimensions for analysis: it is often wise to identify several origin-destination pairs (an O-D pair is the couple of an origin zone and a destination zone) and several time periods (eg. heavily trafficked vs. low trafficked).

The analysis may focus on a given market segment, i.e. an intersection point for a combination of analysis criteria. More often it addresses a group of segments, each of which is considered homogeneous with respect to the criteria. In fact each segment may be further subdivided, eg. an origin or destination zone may be divided into subzones.

A1c A list of state variables

A distinction can be drawn between quantity variables, quality variables and behavioural parameters.

The basic quantity variable is a **trip volume**, i.e. a number of moving entities under specified spatial and temporal conditions. The unit may be a person, a vehicle, a reference weight, a reference value etc. The volume may be disaggregated by demand segment. The spatial condition may be the traversal of a network link, in which case the quantity variable is called the link volume. A more abstract spatial condition is the origin-destination (O-D) pair: an O-D volume is a number of trips from a given origin zone (aggregate of individual locations) to a given destination zone. The temporal condition is often a given hour in the day in an urban context, or the whole day in an interurban context. Elementary time periods are usually considered in an aggregate way, based on an (abstract) average period defined over a given range of elementary periods.

Quality variables are less homogeneous than quantity variables: we may distinguish the economic variable of price and more generally consumer cost, from the physical variable of travel time. The economic significance of the travel time is usually described by the concept of the value of time (VoT), defined as the maximum amount of money which the consumer is willing to pay to save a unit of time. Then the travel time multiplied by the VoT may be added to the transport price, yielding the generalized cost of transport to its consumer. Transport prices and times vary across demand segments, spatial and temporal circumstances.

As transport is basically a move from an origin point to a destination point, the segmentation with respect to O-D pair is of paramount importance. It is conveyed by the two concepts of transport zone and O-D matrix.

Let us define a transport zone: a transport system pertains to a specific area which is a set of individual locations. A transport zone is a subset of individual locations: rather than considering every trip on an individual basis from their own origin place to their own destination place, trips are aggregated with respect to origin zones and destination zones, making up origin-destination (O-D) pairs.

An O-D matrix provides information (eg. volumes, travel times) disaggregated with respect to O-D pair: it contains as many lines as there are origin zones and as many columns as there are destination zones. Every cell contains the information associated with the pair of its origin (line index) and destination (column index).

Demand behavioural parameters. These pertain to consumer classes and describe their economic behaviour. Instances include the value of time, the value of comfort and safety, the elasticity of volume to generalized cost, and more generally parameters of demand functions involved in distribution models of destination choice or in route- or mode- choice models.

Supply behavioural parameters. These describe the economic behaviour of suppliers. As the number of transport suppliers is relatively small compared to the number of consumers, each one may be paid a particular attention in the analysis of a given transport system. This explains for the relative scarcity of models of supply behaviour. Supply behavioural parameters include the actualization rate, the minimum ratio for return on investment etc.

Physical parameters. These pertain to the physical impacts of the transport system, notablu the impacts of traffic. Instances include emission rates by type of pollutant and of motor (also with respect to operating speed), noise emission rates etc.

A2 Overview of transport survey methods

The variety of state variables and of dimensions for analysis results in a complex measurement problem. As it is hardly possible to measure each variable in a complete, accurate way, we can only avail ourselves of partial measurement, restricted to a subset of variables, of spatial conditions and of temporal conditions.

Three broad families of transport survey methods may be distinguished, based on their relationship to the surveyed trips: **network data** are measured along specific network links; **en-route surveys** intercept trips along given links and yield a description of the link users; **off-trip surveys** involve interviewing transport consumers to reveal their usage of the transport system or to state their preferences towards scenarios.

A2a Network data

Network data are associated with specific network links or paths (a path being a sequence of links); they provide an aggregate information on local traffic volumes and trip conditions, with limited possibility of disaggregation apart from spatial and temporal conditions and vehicle type. These observations involve either stationary observers (eg. traffic counter) or moving observers (eg. chase car, floating car).

A2b En-route surveys

An en-route survey consists in the real-time observation of a sample of trips. Depending on the duration of the observation (with interview or not, short or long interview), a pre-specified amount of individual information is obtained. Interviews can reveal the origin and destination places, the activities before and after the actual trip, the number of passengers or type of good.

This yields a description of the link users and of the patterns of usage, useful for customer analysis.

Instances include roadside interviews (for pedestrians or motorway users), on-board or in-station surveys in the case of public transport.

A2c Off-trip surveys

These involve off-line interviewing transport consumers, passengers or shippers or carriers, about their past and future trips. Instances include home-based surveys on the mobility of households, firm-based surveys on the passenger trips with professional purpose or on the firm’s shipments.

An off-trip survey will typically reveal much individual information about a limited number of respondents. The expansion of the survey sample to the population in an important issue. The survey mode influences the contents of the questionnaire: face-to-face interviewsare recommended to introduce sophisticated questions such as choice scenarios. Phone or mail surveys may be appropriate for a limited number of simple questions.

A typical questionnaire comprises five parts: (1) socioeconomic data; (2) description of transport equipments, eg. cars, bikes, subscription to public transport network; (3) description of a subset of trips; (4) mobility habits; (5) explanations about transport choice or self-expression about hypothetic choice scenarios.

Off-trip surveys which emphasize the statement of preferences across hypothetic scenarios are called Stated Preferences surveys. Their results are called Stated Preference data, as opposed to Revealed Preferences data obtained from past, observed choices.

B Network data

The phenomenon of traffic extends over two dimensions, space and time. Its description is eased by macroscopic, aggregate variables, which quantify its intensity (number of vehicles) and efficiency (mean speed).

The lesson aims at defining the basic macroscopic traffic variables and describing the related measurement methods. It is comprised of five parts. Part 1 defines the basic traffic variables (from flow and concentration to time speed and space speed) and states their mutual relationships, either analytic (flow continuity equation) or empiric (speed-flow curve). Part 2 and Part 3 introduce the stationary observer methods and the moving observer methods. The associated measurement devices are briefly reviewed in Part 4. Lastly, Part 5 contains three exercises.

B1 Traffic variables

B1a Defining traffic

Traffic is "pedestrians or vehicles or ships or aircrafts moving along a route". It therefore consists in the movement of traffic particles in the two dimensions of space and time. In traffic analysis particles are called vehicles, without loss of generality. For convenience the space dimension is reduced to the scalar dimension of a route, although most transport modes involve bi-dimensional or tri-dimensional moves (eg. multilane highway, the pavement, the sea, the air).

The two dimensions of traffic and its compound nature make it quite difficult to describe and to quantify. In general, a description of traffic is associated to certain space and time conditions. It basically consists in assessing a number of vehicles and their respective speeds: the number of vehicles measures the quantity of traffic, whereas the distribution of speeds measures its quality, its performance. The measures may be disaggregated by vehicle classes, because the counting of objects only makes sense when the objects are sufficiently homogeneous.

B1b Quantity variables: flow rate and concentration

The fundamental traffic quantity is a number of vehicles $\mathrm{N}_{\text{section}}^{\text{period}}$ associated to certain space and time conditions.

When the space condition reduces to a given point *x* on a network link, the number of vehicles, $\mathrm{N}_{x}^{t,t+\Delta t}$, which pass through the point during a certain period $]t, t+\Delta t]$ is called the flow, or volume, during that period. The **flow rate** *q* at point *x* is the ratio of the flow to the duration of the period: $q = \mathrm{N}_{x}^{t,t+\Delta t} / \Delta t$.

When the time condition reduces to a given instant *t*, there is a given number of vehicles, $\mathrm{N}_{x,x+\Delta x}^{t}$, located on a given route section $]x, x+\Delta x]$. The **concentration** (or density) *k* at instant *t* is the ratio of that number to the length of the section: $k = \mathrm{N}_{x,x+\Delta x}^{t} / \Delta x$.

The definition of flow and concentration may apply to passengers (or load) as well as to vehicles. This gives rise to the intermediary notion of mean vehicle occupancy rate, defined as $\mathrm{N}_{\text{section}}^{\text{period}}(\text{Load units})/\ \mathrm{N}_{\text{section}}^{\text{period}}(\text{Vehicles})$.

B1c Speed and travel time

Although the instantaneous speed of a given vehicle is unambiguous ([1]), there are two definitions of mean speed, depending on the space and time conditions.

The mean **time speed**, also called mean spot speed and denoted by $\mathrm{v_T}$, corresponds to one point *x* and a given time period $]t, t+\Delta t]$. It is the mean speed over the vehicles passing *x* from *t* to $t+\Delta t$. The time speeed distribution represents the speeds of vehicles passing point *x* during the period.

The mean **space speed**, denoted by $\mathrm{v_S}$, corresponds to one instant *t* and a given section $]x, x+\Delta x]$. It is the mean speed over the vehicles located in that section at time *t*. The space speed distribution represents the speeds of all the vehicles on the section at that instant.

B1d Vehicle classes

A number of applications require to disaggregate traffic measures with respect to vehicle type: traffic legislation and regulations (different vehicles have different driving conditions), accident analysis and prevention (eg. an on-fire truck more dangerous in a tunnel than an on-fire car), traffic effort on pavement and bridges, traffic capacity and operation, parking system design.

A typology of roadway vehicles may be: bicycles, motorbikes, motorcycles, motor cars, station wagons, utilities, panel vans, rigid/articulated freight carrying trucks, other trucks, buses, tractors, plants, caravans and trailers.

Even with this classification there may be significant differences within a type, especially with respect to speed.

B1e Analytic relationships under stable flow

Classification with respect to speed enables one to derive relationships between time and space variables, under the assumption of **stable flow**, i.e. on a given section and during a certain period every vehicle maintains their own speed.

Let us denote by *m* a speed class with speed v_m, flow rate q_m and concentration k_m. By aggregation over speed classes, we obtain that $q = \sum_m q_m$ and $k = \sum_m k_m$. From the definition of the mean time and space speeds, $q\,\mathrm{v_T} = \sum_m q_m v_m$ and $k\,\mathrm{v_S} = \sum_m k_m v_m$.

From the stable flow assumption, $q_m = k_m v_m$ because the vehicles with speed v_m passing *x* from *t* to $t+\Delta t$ are the same ones which were located on section $[x-v_m\Delta t, x]$ at time *t*, hence $q_m \Delta t = k_m v_m \Delta t$. Then $q = \sum_m q_m = \sum_m k_m v_m = k\,\mathrm{v_S}$ which is the **traffic flow continuity equation**.

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[1] Within a given refential

Aggregation in space yields that $k = \sum_m k_m = \sum_m q_m / v_m = q \sum_m (q_m / q)(1/v_m)$ hence $k/q = 1/v_S = \sum_m (q_m / q)(1/v_m)$: the mean space speed is the time harmonic mean of the speed.

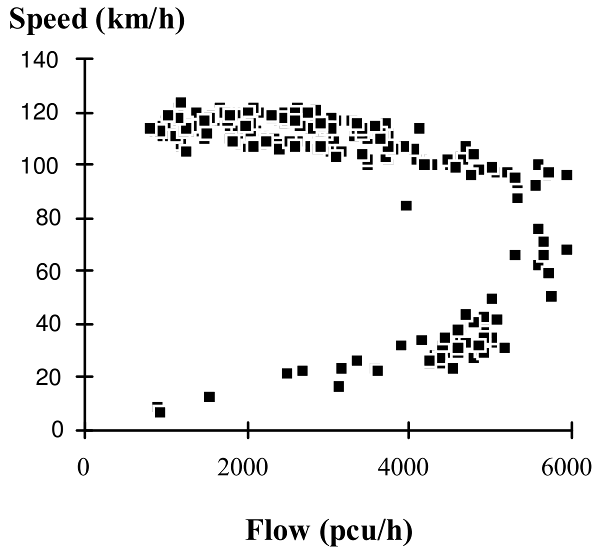
Aggregation in time yields that $v_T = \sum_m (q_m / q) v_m = [\sum_m (k_m / k) v_m^2]/v_S$: in the distribution of space speeds $v_S . v_T = E[v^2] = \mathrm{var}(v_S) + v_S^2$. This is Wardrop's formula, which implies that the mean time speed is superior or equal to the mean space speed. There is equality iff all vehicles have the same speed.

B1f Empiric relationships for uninterrupted flow

Uninterrupted flow occurs in a traffic stream unaffected by external factors (such as junctions and intersecting streams or change in the number of lanes). It is subject to internal interactions, called **congestion**: the higher the concentration of vehicles along a given section, the more difficult it is to overtake a slower vehicle. When concentration is very high the traffic experiences stop-start conditions with reduced speed and flow. When concentration is very low the traffic stream is under free-flow conditions: each vehicle can maintain an uninfluenced free flow speed.

This empiric behaviour of traffic is captured into a flow-speed diagram, which contains joint observations of the mean speed and the flow rate for a given section and a set of time periods of equal duration. In a speed vs. flow plot, the observation points define a horseshoe diagram, in which the upper arm is the uncongested part from null flow and free flow speed to a limit flow rate called capacity. In the near capacity part, the flow is approximately constant whereas speed may vary considerably. This corresponds to traffic in which the vehicles are quite close to each other but can still move on without stopping most of the time. The lower arm is the congested part, in which both speed and flow are reduced. This corresponds to traffic in which the vehicles are very close to each other and forced to stop during a large proportion of the time.

Fig. a. Speed versus flow.



Related diagrams of mean speed vs. concentration and flow rate vs. concentration (proportional to occupancy rate) may also be illustrated. Their common advantage over the speed-flow diagram lies in the (approximately) deterministic behaviour of the speed or the flow with respect to the concentration. The flow-concentration diagram has also a horseshoe shape, with maximum flow value attained at the critical concentration. The speed-concentration diagram exhibits a monotonic decreasing curve.

The values of the free flow speed, capacity flow rate and critical concentration are particular to a given section: they depend on the geometric design, the speed limit, the overall weather conditions, the traffic composition etc. The "complete" diagram, with three distinct regions, can only be observed on sections which experience sufficient concentrations.

Fig. b. Flow versus occupancy rate.

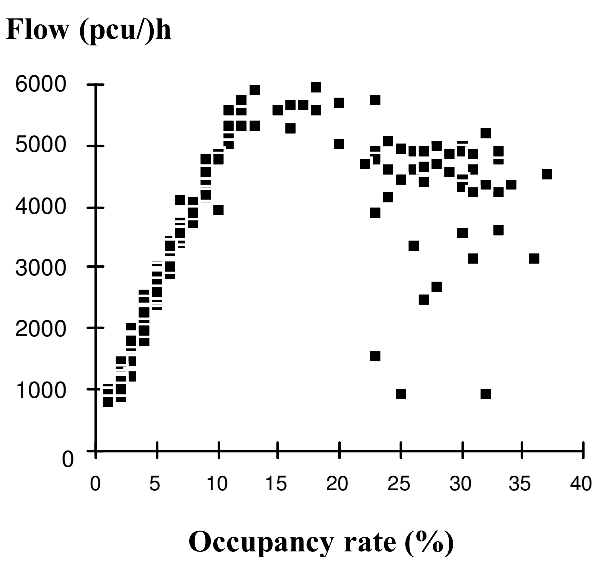
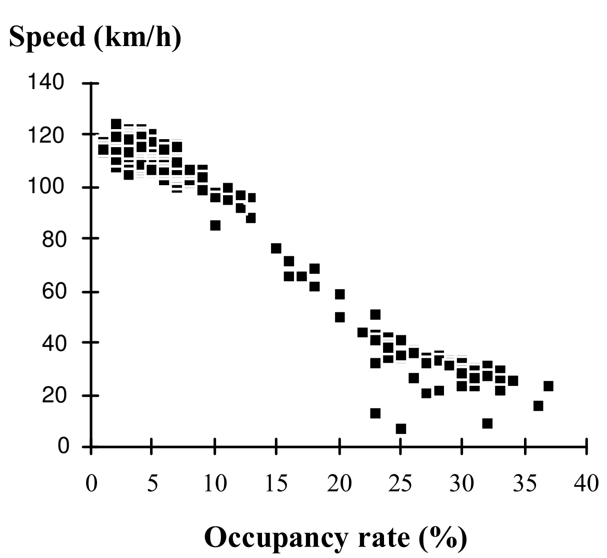


Fig. c. Speed versus occupancy rate.



B2 Measurement by stationary observer

A stationary observer is a human or device located at a fixed point, as opposed to the movement of traffic. The stationary observation is often limited to only one point along the section: however an elevated observer (eg. aerial photography) may cover a large section.

B2a Vehicle counting

A vehicle counter can measure the number of vehicles passing the measurement point during a certain period, i.e. the flow. Identification of vehicle type requires as many individual counts as there are types.

A proxy for vehicle counting is axle counting (an axle is a wheel pair). The number of axles must be divided by an average axle rate to approximate the number of vehicles.

Concentration can be measured directly by an elevated observer, by localization of individual vehicles along the section.

B2b On the aggregation of vehicle types

The identification of vehicle types requires to multiply the counts. When this is not feasible, only the total number of vehicles can be measured and the composition of traffic is not known. When vehicle types are identified, additional aggregate traffic variables can be expressed with respect to a given type. In roadway traffic, the passenger car is used as the typical traffic unit: heavy vehicles of a given type are converted into passenger car units (p.c.u.) based on an average equivalency coefficient which depends on the road design (number of lanes, slope, curvedness). On a straight section with no slope, a typical value of 2.0 or 2.1 serves as the equivalency coefficient of trucks to cars.

This conversion rule also yields flow and concentration in passenger car unit.

B2c Concentration from local occupation time

Some stationary devices can measure the occupation time of the measurement point, i.e. the time during which point x is occupied by a vehicle.

Let Δt be the duration of the measurement period. The occupation time $\Delta\tau$ decomposes onto the individual occupation times $\delta\tau_i$ of the n vehicles i passing point x during that period.

Assuming that vehicle i has length ℓ_i and that the device has a residual length ℓ', the individual occupation time $\delta\tau_i$ corresponds to the distance $\ell_i + \ell'$ hence to an individual speed of $v_i = (\ell_i + \ell')/\delta\tau_i$. If the distribution of individual vehicular lengths and speeds are independent, then $\Delta\tau = \sum_i \delta\tau_i = \sum_i \frac{\ell_i+\ell'}{v_i} = (q\Delta t)\,\mathrm{E}[\frac{\ell+\ell'}{v}] = q\Delta t(\mathrm{E}[\ell]+\ell')\,\mathrm{E}[\frac{1}{v}]$ hence $\frac{\Delta\tau}{\Delta t} = q(\mathrm{E}[\ell]+\ell')\frac{1}{v_S} = (\mathrm{E}[\ell]+\ell')k$. Thus concentration k may be recovered from $\frac{\Delta\tau}{\Delta t}$ and $\mathrm{E}[\ell]+\ell'$.

B2d Local measurement of speed

The spot speed of a vehicle can be directly measured by means of a radar, based on the Doppler effect (the frequency of a wave emitted by the radar changes with respect to the relative speed of the vehicle, this change can be measured when the wave comes back from vehicle to radar[2]).

More often the spot speed is indirectly measured as the quotient of a travelled distance by the corresponding travel time. This principle is used when dividing the sum of the vehicle length and the device residual length by the occupation time. A more accurate procedure is to combine two local devices with intermediary distance of δD, and to measure the individual differences in passing times, δt_i. Then $\delta D/\delta t_i$ is equal to the mean speed of the vehicle between its two passing times.

Another instance involves an elevated observer to locate the vehicle positions at two close instants t_1 and t_2: denoting by δd_i the distance between the two successive positions of vehicle i, the ratio $\delta d_i/(t_2 - t_1)$ indicates the mean speed of the vehicle from t_1 to t_2.

B3 Measurement by moving observer

In a moving observer method, the observer takes part to the traffic stream. This involves either a floating vehicle, of which the driver attempts to travel at the mean space speed, or a chase vehicle, of which the driver attempts to copy the movement of a selected vehicle. Both cases require prior definitions about individual trips.

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[2] denoting by θ the angle from the route axis to the radar axis, and by Δv the relative speed of the vehicle with respect to the radar, the change in frequency is $\Delta F = 2v\cos(\theta)/\lambda$ where λ is the signal wavelength

B3a Definitions for individual time and speed

There are several notions of individual time on a given section. Each of them provides a related definition for mean speed, on the basis of $v = L/T$ where L is the sectional distance, T the particular time and v the related mean speed. Here mean speed refers to an individual vehicle and to a time interval.

Free flow time is the time required by an unimpedded vehicle to traverse the section.

Travel time is the actual time taken to traverse the section. It splits into **stopped time** (during which the vehicle is stationary) and **running time** (during which the vehicle is in motion). The running speed is often distinguished from (travel) speed, also called commercial speed in public transport.

Delay is the difference between the travel time and the free flow travel time.

B3b Floating car strategy

The floating vehicle method applies to long road sections. Although the basic result is the mean travel time of the surveyed section, the method also provides information about stopped time, running time, number of stops and flows.

The mean travel time is obtained in the following way. The floating vehicle measures its own travel time in traffic direction, T_d, and also the number of overtaking vehicles, n_{d+}, and the number of overtaken vehicles, n_{d-}. These are related to the sectional distance L, flow rate q_d and concentration k_d on the basis of $(n_{d+} - n_{d-})/T_d = q_d - k_d L/T_d$. Knowing q_d this yields $k_d = (T_d q_d + n_{d+} - n_{d-})/L$ hence $\mathrm{v_S} = q_d / k_d$ and $T = L/\mathrm{v_S}$.

If no prior measure of q_d is available, another floating vehicle travelling in the return direction of traffic yields a number n_r of vehicles encountered in the original direction and a travel time of T_r: it also holds that $n_r / T_r = q_d + k_d L / T_r$. On combining the two relationships, we obtain $q_d = [n_{d+} - n_{d-} + n_r]/(T_d + T_r)$ and $T = T_d - (n_{d+} - n_{d-})/q_d$.

Both relationships are established by decomposing the traffic into speed classes m. Vehicles of class m have relative speed $v_m - L/T_d$ with respect to the test vehicle, which is "overtaken" by a net number of $n_{d+}(m) - n_{d-}(m) = T_d k_d(m)(v_m - L/T_d)$ vehicles of class m during T_d. Aggregation over speed classes yields that $n_{d+} - n_{d-} = T_d q_d - L k_d$.

B3c Chasing car strategy

The chasing vehicle method applies to long road sections. The objective is to collect a sample of individual travel times, stopped times, running times. The sample yields empiric mean and variance estimators.

The method involves random selection of vehicles in the traffic stream, for instance by choosing the first vehicle to pass a reference point after a randomly selected instant. A difficulty arises when the chased vehicle leaves the section at an intermediary point: if the sampling unit is the vehicle then the chaser should continue and pick up a nearby vehicle to follow, whereas if the sampling unit is the route then the run should be aborted and the chaser should return to the begin point to pick up a new vehicle. Another difficulty arises when the chased vehicle infringes the speed limit.

A chasing vehicle may also be used to collect continuous information on the chased trip, i.e. the continued monitoring of instantaneous position and speed. The record may be used to simulate the trip in dynamometer testing in order to simulate fuel consumptions or pollutant and noise emissions.

B4 Traffic measurement devices

A traffic measurement device is a sequence of a sensor that emits raw data on vehicle arrivals, a detector to receive, amplify or transform the signal, a recorder (data logger) and a processor to interpret the signal, connected together by transmission equipment.

We shall briefly review the devices in common use: human counting, pneumatic tubes, inductive loops and video camera. Aerial photography, despite its efficiency to measure concentration and space speed, is not described because of its high cost. Lastly we shall allude to instrumented vehicles.

B4a Human counter

A human observer is able to detect and interpret visual, audile and even olfactory signals. He has not yet been matched by machines as regards recognition. His limits pertain to the accuracy (eg. arrival times), the duration of measurement (because fatigue increases the risk of omission, of misinterpretation and of misrecording), and to the traffic intensity which may exceed the capacity of the observer.

B4b Pneumatic tubes

A pneumatic tube detector is a thick-walled rubber tube, stretched across the road or a number of traffic lanes, held in position by clamps nailed into the pavement. It is connected to a diaphragm air switch on the edge of the pavement. When crossed by a wheel, a pulse is created and sent to the diaphragm, whose vibration closes an electrical circuit and produces an electrical signal sent to a recorder.

The low cost, robustness and portability of pneumatic tube recommend it for temporary use. It is restricted to axle count and lanes adjacent to the road shoulder, but astute combination of several tubes can alleviate these limits.

B4c Inductive loops

The inductive loop operates by monitoring the electrical properties of a coil of wire, buried just below the road surface or mounted on it. An alternating current flows through the coil, which generates an alternating magnetic field. The presence of an electrical conducting body, such as a motor vehicle, over the detector alters the field, which induces additional currents in the coil and changes the inductance. The degree of change depends on the metallic mass and its very position relative to the coil.

Loops are usually located in a traffic lane, with one separate loop in each lane. There is a trade-off between sensitivity which requires in-breadth extension, and accuracy which is impaired when one vehicle is detected by two adjacent loops. A typical loop length (denoted as ℓ' in detection of occupation time) is about 0.80 m. Shorter loops ($\ell' \approx 0.2$ m) allow to reconstitute the profile of axles for a long vehicle.

One loop can detect vehicles and measure occupation time. A serial combination of two loops reveals an elementary vehicle travel time, hence spot speed and also vehicle length.

B4d Video camera

Video recording of traffic at an intersection or on a length of road is best performed from an elevated camera position. Every image amounts to a set of detection points, with particular luminance level. Adjoining points with similar luminance levels usually correspond to the same object, which can be recognized either by a human or an algorithm for artificial vision.

The image reveals the presence of vehicles on a given road section, together with the type, number plate and length (by comparison to reference lines).

A sequence of images, analyzed by vehicle tracking, also reveals the individual spot speeds, the flow, the concentration, the occupation time and the occupancy rate.

However the video camera detection has limits, due to lightning conditions, height of certain vehicles etc.

B4e Test vehicles

A test vehicle used for floating car or chasing car surveys typically consists in a car with one driver and one observer. The observer monitors his observations either on paper or dedicated records.

Modern technology will probably reduce the need for car following, owing to localization devices (ex. Global Positioning System) which can be delivered to drivers for the duration of a survey.

B5 Exercises

B5a Macroscopic variables from microscopic data

The following table indicates the respective positions of vehicles on a road section at two instants $t_1 = 0s$ and $t_2 = 0.10s$. Each position is measured by the distance in meters from a reference point.

Vehicle	Position at t_1	Position at t_2
1	25	33
2	36	42
3	45	57
4	53	66
5	114	123
6	162	175
7	188	195
8	213	219
9	240	243
10	285	294

1. Which measurement device(s) can yield such data?
2. Evaluate the individual spot speeds.
3. Evaluate the mean space speed and the concentration, assuming the section is 300 m long.

4. The flow is measured at point x distant by 1,500 m from the reference point. The measurement period begins at $t = 5s$ and lasts 15 s. Evaluate the flow rate and the mean time speed at point x (hint: for each vehicle, determine its position at times t and $t+\Delta t$). It is assumed that each vehicle maintains its spot speed as of time t_1.

5. Compute the empiric variances of the space speed and the time speed.

B5b Measurement from inductive loop

A double induction loop produced 10 successive hourly observations, as indicated in the following table.

Hour	Cars	Trucks	Occupancy rate (%)
1	595	105	2
2	1278	142	4
3	3422	116	10
4	5504	107	17
5	5018	73	29
6	5915	95	19
7	5790	120	18
8	5897	158	18
9	5684	141	16
10	5947	132	17

1. Compute the total vehicular flow rate and the proportion of heavy vehicle for every hour.
2. Based on an equivalency coefficient of a truck to passenger cars of 2.1, compute the hourly flows in p.c.u.
3. Plot the hourly p.c.u. flows against the occupancy rate.
4. Recover the concentration from the occupancy rate, assuming a loop length of 0.80 m, an average car length of 4.3 m and an average truck length of 8.5 m.
5. Based on the p.c.u. flow, the concentration and the flow continuity equation, recover the mean space flow.

B5c Travel time measurement

As a member of a consulting firm, you are asked to measure the mean travel time on a road section with length L = 3.0 km. For each of the following methods, please put forward a survey scheme, including sample size. Evaluate the accuracy of result with respect to sample size and the survey costs in terms of required workers and equipment and duration of use:

1. Floating car method.
2. Chase car method.
3. Stationary observers at each endpoint of the section.

C Instances of en-route surveys

C1 An instance of roadside interviews

In France, the national administration is responsible for the main interurban roadway links, i.e. the freeways and national highways. Although the operation of most interurban highways has been delegated to motorway operators (all of which state-owned, apart from Cofiroute), the administration is in charge with the operation of national highways and the planning of both freeways and national highways.

Its traffic studies are performed by seven state consulting firms, the Centers for Technical Studies in Building, Transportation and Housing; the French acronym reads CETE. Each CETE is in charge of a specific area, for which it maintains origin-destination matrices of car and truck trips.

The O-D flow information is collected through a series of roadside interview surveys on the major links, with standard questionnaire forms for either cars or trucks.

We shall present this global interception survey by first describing the location of interception sites and the administration procedure. Then we shall introduce the questionnaire and review the post-survey data management. Lastly we shall outline the statistical processing.

C1a Location and administration

Of the two attached maps, the first one localizes the Poitou-Charentes region whereas the other displays the major interurban roadway links in that region. It also depicts the towns of national or regional significance and the sites of the interception surveys.

Location. Most local interview surveys are oriented, i.e. their scope is limited to one direction of traffic. Some of them deal exclusively with either car or truck traffic. The sites are selected to provide a coverage of the O-D trips between the major regional cities and also between these and the "rest of the world".

Temporal conditions. As of current practice, each local interception is a one-day survey for a weekday, preferably a Tuesday or a Thursday, selected in an out-of-summer month. There are some summer surveys, mostly implemented in touristic areas. Only the daylight period of the day is surveyed (about from 7 am to 7 pm), because it accomodates the vast majority of traffic and also for the sake of surveyors' safety.

Administration procedure. Each interception site is selected very close to a parking area. The whole link trafic is slowed down at the approach of the parking area by policemen, whose contribution is compulsory in France to stop the traffic. The policement randomly select the sampled cars and trucks, which are directed to the parking area. A typical relevant parking capacity is of 12 cars or 6 trucks. The driver of each selected vehicle, under the priority rule First In-First Out, is interviewed by a surveyor who reads and explains the questions and writes the answers.

Of course the cooperation of the police greatly enhances the response rate. A typical number of surveyors if 3-6 persons at work simultaneously, which amounts to 5-8 surveyors for the working day. The typical duration of an interview is about two minutes.

Fig. d. Map of French regions.

Fig. e. Roadway en-route surveys in the French region Poitou-Charentes.

En-route surveys are located (approximately) by black triangles.

C1b Objectives and contents of the questionnaires

The basic objective is to provide the two O-D matrices of car and truck trips, which may disaggregated with respect to time-of-day and trip purpose (i.e. passenger activity, type of truck or type of good).

General information. The surveyor notes the type of vehicle and the survey time (hence the approximate passing time). Every driver, of either a car or a truck, is asked about his origin and destination places. The elementary location unit varies from the inner study area to the outer area. In France the basic urban unit is called the commune; it possesses its own municipality. There are 36,000 communes in metropolitan France, for about 60 million persons on roughly 550,000 square kilometers. Inside the study area, the commune is the elementary location unit in rural areas, whereas large urban agglomerations may be subdivided into several units (eg. 20 zones for the Bordeaux agglomeration which accomodates about 700,000 inhabitants). Outside the study area, the location unit becomes larger and larger as the distance from study area increases: neighboring countries are taken into account by one or a few zones (eg. one zone for Switzerland, four for Germany). Associated with each location unit is a specific code, written by the surveyor on the questionnaire form.

General information also includes trip frequency.

Specific passenger information. The driver is asked about the number of persons in the car, the departure time, the reasons for using the car, the origin and destination purposes i.e. the previous and next activities. The typical list of passenger activities includes Home, Work, Personal Business and Leisure.

Specific good information. The driver is asked about the type of good, the weight of load, the type of truck, the origin and destination purposes i.e. the previous and next loading or unloading places.

C1c Data management

There are two coding steps for each interview: the first one takes place when the surveyor records the answers on the questionnaire form, the second one consists in putting the information into a database, owing to a special purpose software. The first coding step lasts about two or three minutes, whereas the second lasts about one minute.

The results of each local survey are collected in a specific database, which includes not only the coded responses but also general information about the location and date of survey, the number of surveyors, the total traffic and sample size (disaggregated by vehicle type).

All databases are managed under a special purpose package named GEODE. GEODE enables the analyst to aggregate the results of several local surveys to obtain an O-D matrix, to visualize the map of the study area and the databases in common spreadsheet format. It also includes a function for evaluating the variance of any O-D link proportion (i.e. the proportion of ling traffic which belongs to a given O-D pair).

The resulting O-D matrices are symmetric, two-way matrices since most local surveys are one-way and have no reverse-way counterpart.

C1d Statistical analysis

The **sample rate** of a local survey is obtained as the ratio of the sample size to the link count during the survey period. It may be disaggregated by time-of-day. This yields an individual weight associated to each observation. A better solution would be to measure the sample rate as the ratio of sample size to average link count (evaluated as the empiric mean in a sample of several days).

Every **O-D trip rate** is evaluated as the aggregation of the predicted link O-D trip rate over all surveyed links in a screenline separating the origin from the destination. This requires the existence of a such screenline (unsignificant links need not be surveyed). When several screenline fulfil the condition, one of them is selected by the analyst, maybe not at random but with no statistical consideration. A relevant statistical rule would be to select the screenline which yields the least total variance on the O-D matrix rate.

As regards **variance** of an O-D flow and **covariance** of two O-D flows, no evaluation is made. We shall provide several solutions in the third statistical lesson.

C2 Public transport surveys

D Principles of interception surveys

Roadside interviews are a particular instance of local interception surveys. A local interception means the observation of a trip along a given network link, whatever the level of detail of the observation, the type of trip or the transport mode. Therefore local interception surveys also include link traffic counts, station-based and on-board surveys.

A local interception survey may also be an element in a system of interception surveys, let us say a global interception survey, in which observation along several network links yields an O-D matrix between observed links.

This lesson reviews the main techniques of local and global interception surveys in order to assess their respective scope, risk of error and cost. It comprises two parts: first an overview of the techniques, secon an evaluation of their potential relevance for O-D matrix constitution.

D1 Techniques for interception surveys

Every interception technique falls into one out of two categories, depending on whether the measurement requires to intercept a given trip once or twice. A double interception involves a matching technique.

Single interception may be accomplished by mere local observation or en-route interview (either roadside or on-board or in-station). Double interception may be performed by matching of number plates or of windscreen stickers, or owing to headlights.

D1a Mere local observation

Apart from some non-visual devices (eg. inductive loops), local observation is a visual one which involves either a human observer or a video camera. In both cases a monitoring device is required: pen and paper or tape recorder in the case of a human, a film in the case of a camera. The observation must be processed prior to analysis, which necessitates a processing device which is either a human or a computer.

The scope of mere observation is restricted to visual, external attributes of the vehicles: their existence, the number of passengers, the category from within a small subset, the number plate in the case of a motorway vehicle, the turning movement at the nearest junction.

Visual observation requires legibility hence good lightning and weather conditions. The potential errors are associated with omission, misrecording or misprocessing. The omission and misrecording rate of an observer o may be measured in the following way. Let us assume that there are three observers n, o, p stationed along the network link, with o in middle position. The accuracy rate $A(o)$ of observer o can be defined as the ratio of the number of matches between n, o and p, to the number of matches between n and p.

As regards survey cost, measurement costs contribute the major part. Processing costs may be amortized by use of computers and specialized software.

D1b En-route interviews

The definition encompasses a variety of instances: roadside interviews, on-board or in-station surveys for the public transport of passengers. In all cases the questionnaire aims primarily at the origin and destination places and the type of trip.

An origin or destination place may be defined as an activity location for car drivers and long-distance passengers, or as an access or egress station for short-distance passengers (eg. bus).

Type of trip may refer to the mode, the time-of-day, the frequency, the number of persons or load weight, the previous and next activities for passenger or the type of good.

Roadside interviews may include up to about ten questions. On-board bus surveys may be limited to statement of purpose, access and egress points. Interurban on-board (in the train or on the plane) and in-station surveys may be much longer and include questions about the household, the person, the mobility practice (see off-trip surveys).

The risks of error are associated with non-response, sampling error, erroneous response and misrecording. Survey costs consist in observation costs (two thirds, proportional to time spent) and processing costs (one third), provided that a standard questionnaire be used.

D1c Number plate matching

The number plate matching involves stationing observers at selected network points in and around an area, to record the vehicle type, number plate and passing time. Matching the local observations yields point-to-point O-D trip matrices by vehicle type, and also travel time O-D matrices.

The number plate may be only partially observed and recorded.

Local number plate observation amounts to a mere local observation and is plagued with the same risks of error. Additional potential errors pertain to missynchronization of local clocks and to data matching (eg. error in number plate or vehicle type). The latter may be partially alleviated by specialized post-processing.

Observer error on the number of trips from point *i* to point *j* may be corrected by replacing the number of matches $N_{i \cap j}$ with $N_{i \cap j} / [A(i)A(j)]$, in which *A(o)* is the accuracy rate of observer *o*.

An advantage of camera observation lies in post-processing: human examination of the film will ensure a high quality identification of number plate and vehicle type, eventually on the basis of other vehicle attributes.

Survey costs depend on the use of cameras and computers. An integrated system with camera observation and recording, human identification and computer post-processing will mainly involve the amortizement costs (eg. rental costs) of cameras and the time spent on identification.

D1d Colour sticker and headlight techniques

These may be analyzed as simplified from number plate surveys. A sticker survey involves issueing stickers at selected points called origins, with sticker colour specific to the origin, and reading or collecting stickers at "destination" points.

Stickers can be placed on the windscreen of a stopped or parked vehicle, or handed out to a driver or a passenger in the case of public transport.

A sticker technique is appropriate when there are a limited number of origin and destination points.

A headlight survey concerns one origin point only, at which point drivers are asked to switch on their headlights until end of trip or a given duration has elapsed (eg. 20 minutes). Observers stationed at other selected network points will count two numbers of vehicles, with headlights on or off.

D2 Potential contribution to O-D matrix

D2a Comparison of techniques

The relevance of interception techniques to contribute to an O-D matrix depends on conditions specific to each technique. Under proviso that each technique should meet its own requirements, the ranking in order of decreasing value is as follows: first en-route surveys, second number plate surveys, then colour sticker and headlight surveys, lastly link counts.

En-route interviews provide results closest to an O-D trip matrix, since the exact trip origin and destination places are explicitly identified. Their relevance requires sufficient sample sizes and a sufficient network coverage.

Number plate surveys yield partial O-D information as regards the individual trips. Individual travel time must also be considered to distinguish a single, continuous trip from a compound trip with an intermediary activity inside of the cordon.

Colour sticker and headlight surveys may be considered as degenerate forms of number plate surveys, resulting in poorer information.

Lastly link counts provide no specific O-D information, unless analyzed with additional information on O-D link proportions. Such information may derive either from more sophisticated surveys, or from a traffic assignment model.

D2b Guidelines for cordon design

A cordon is an abstract boundary isolating an inner area from the outer area. An instance of cordon is a closed ticketing or tolling system, in which each passenger or driver must take or punch a ticket when entering a subnetwork with limited access, and then hand it out or punch it again when egressing the subnetwork. This yields accurate cordon trip matrices (in the absence of cheating) and also travel time O-D matrices if passing times are monitored.

A cordon induces a distinction between:

(1) external trips, which take place in the outer area and do not cross the cordon.

(2) Through trips with outer origin and destination places but also an inner intermediary part.

(3) Exchange trips with external origin (resp. destination) and internal destination (resp. origin).

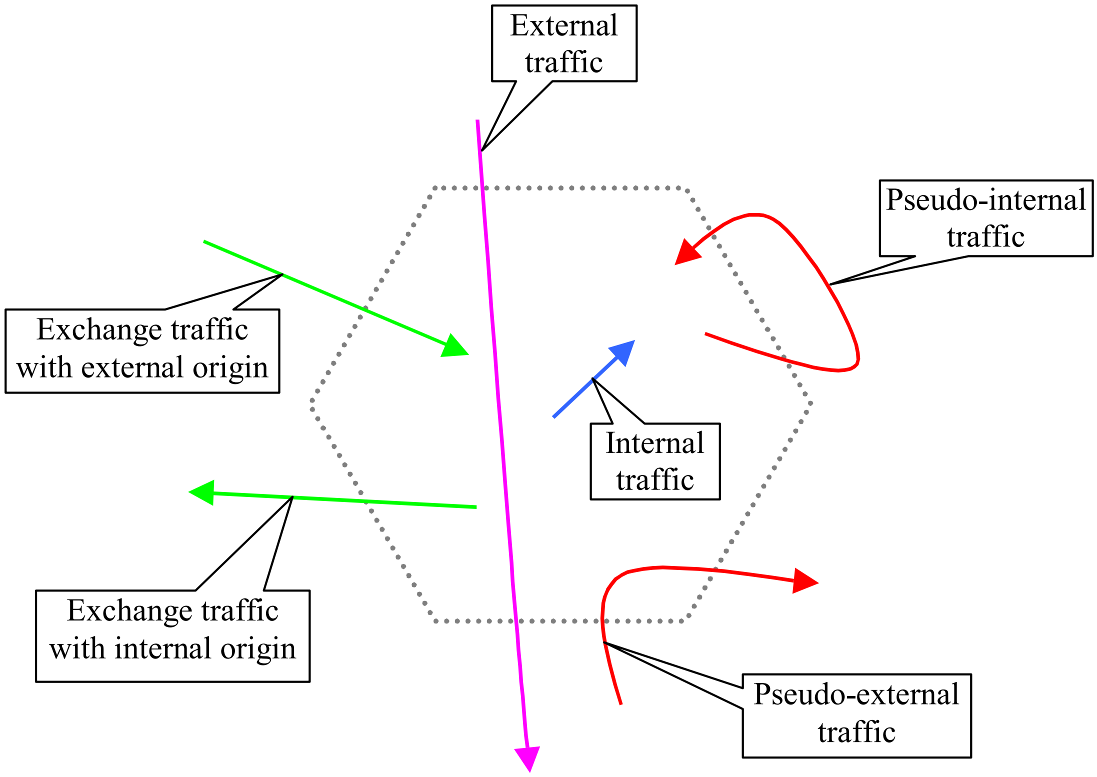
(4) Internal trips restricted to inner area and which do not cross the cordon.

(5) Pseudo-internal trips with inner origin and destination but also an outer intermediary part.

When the survey is aimed at through and exchange trips, the cordon design should avoid as much as possible the pseudo-internal trips and the unsignificant through trips with a very minor inner part. Another obvious, more general requirement is to minimize the number of interception sites. In the case of interceptions located at junctions, it is preferable to observe each turning movement independently (i.e. with a specific device).

The overall trade-off consists in choosing an inner area sufficiently large to avoid pseudo-internal and unsignificant through trips, yet sufficiently small to yield useful information on exchange and significant through trips.

Fig. f. Types of trips.



E Model estimation of O-D flows

The estimation of an O-D trip matrix may be based on various kinds of observations and is a difficult statistical problem. A good solution is to model the O-D flows as the outcome of a demand function, of which the arguments are either parameters or attributes of the origin and destinations and of the transport network. Then the estimation problem pertains to the parameters of the demand function, which enables the analyst to assess the specification error and to transfer some results.

The lesson is made up of four parts. First we shall introduce the demand function and explain its significance. Then we shall briefly describe the appropriate statistical treatment, from probabilistic setting of observations to computation of estimators. Next we shall indicate how to assign the estimation results onto an associated transport network. Lastly we shall comment on specialized software.

E1 On demand function

E1a General

A trip distribution model outputs trip volumes for O-D pairs, based on economic principles together with attributes of the origin and destination zones and of the transport system. The objective of knowledge is twofold: firstly to describe the O-D volumes, which amounts to an empiric knowledge; secondly to explain them, i.e. an economic knowledge.

The problem of empiric knowledge might be solved by multiplying the observations, but this would be much too expensive. A second solution is to use general inference principles, among which maximum entropy: this is a practical solution with also some theoretical value since the principle is explicit. However an inference principle is only a default recourse, with some statistical value but no economic outreach. The third and last solution consists in an economic model based on explicit principles such as rational behaviour. These economic principles provide the only theoretical foundation for transfer over space and time.

E1b Generic formula

Our objective is not to establish economic principles for trip distribution models. We assume that appropriate principles are available and yield a demand function linking inputs to outputs under the following generic formula

$$q = (q_i)_{i \in I} = \tilde{q}(\Theta, X)$$

in which I is the set of O-D pairs indexed by i, q is the matrix of O-D trip volumes, vector X includes attributes of the origin and destination zones and also of the transport system, $\tilde{q}$ is a mathematical function and Θ a vector of parameters.

Our objective is to explain how to apply a demand function to a given case, based on the available observations. Thus, to obtain the O-D trip volumes q_i which are final outputs, we first specify a function $\tilde{q}$ and then estimate the parameters Θ which are intermediary outputs.

E1c The gravity model as an instance of demand function

The gravity model for trip distribution is based on a physical analogy with Newton's law of gravity in mechanics. In the basic form of the gravity model, every O-D flow is proportional to the production of the origin zone, to the attraction of the destination zone and to the reciprocal of the squared distance:

$$q_i = KP_o A_d /(d_i)^2 ,$$

in which *K* is a constant of proportionality, P_o is the production of origin zone *o(i)*, A_d the attraction of destination zone *d(i)* and d_i the distance from zone *o* to zone *d*.

By extension, a formulation of O-D flows as a product of variables related to the origin and destination zones or to the O-D movement is also called a gravity model. This includes the following log-linear model

$$q_i = K \exp(\sum_n \alpha_n X_{n,o} + \sum_m \alpha'_m X'_{m,d}) f(G_i),$$

in which the variables $X_{n,o}$ are origin attributes, $X'_{m,d}$ destination attributes, α_n and α'_m are parameters, G_i is the generalized cost from origin to destination and *f* is a deterrence function (often a decreasing one).

A deterrence function is often used together with **margin constraints**, such as zonal productions and attractions. In a singly constrained model, only the zonal production (or attraction) margins are fixed, whereas in a doubly constrained model both production and attraction are fixed. The margin constraints determine the coefficients π_o and α_d in the O-D flow formula:

$$q_i = P_o \, \pi_o \, A_d \, \alpha_d \, f(G_i) .$$

The gravity model possesses some intuitive, aggregate **economic interpretation**, as the O-D flow increases with respect to production and attraction attributes and decreases with respect to transport cost.

E2 From observations to estimators

E2a Observations

Most available observations consist in traffic counts, i.e. partial sums or linear combinations of O-D flows:

$$x_a = \sum_{i \in I} p_{ai} q_i ,$$

in which *a* denotes an observation (often a network arc) and p_{ai} is the proportion of the i-th O-D pair which contributes to the a-th traffic flow.

This definitional equation involves an assumption of temporal correspondance between the O-D flows and the counted flow. It relates the R.V. x_a to the R.V. q_i and p_{ai}.

As the observation is subject to observation errors, sample fluctuations and temporal variations, the equation is appropriately refined into the following

$$x_a = \sum_{i \in I} p_{ai} (\tilde{q}_i + \eta_i) + \varepsilon_a ,$$

in which the demand function $\tilde{q}_i$ added to the specification error η_i constitutes the mean O-D flow $\mathrm{E}[q_i]$ and the random residual ε_a includes sample fluctuations and temporal variations.

Few estimation models make a clear distinction between the specification error η_i and the residual error ε_a, although their respective significances are quite different. The specification error η_i pertains to the mean O-D flow $\mathrm{E}[q_i]$ hence it is independent from temporal variations. The residual error ε_a pertains to the temporal variability in x_a and $q_i - \mathrm{E}[q_i]$, and also to the observation error and to the uncertainty in the proportions p_{ai}.

When the specification error is not identified, it is easy to derive the PDF function of an observation x'_a from assumptions on ε_a: denoting by f_a the PDF of ε_a, the PDF of x'_a is $f_a(x'_a - \sum_{i\in I} p_{ai}\tilde{q}_i)$ which involves the value x'_a, the parameters in $\tilde{q}_i$ and those of ε_a included in f_a.

A common assumption is that the traffic count is a Poisson R.V. with mean intensity $\tilde{x}_a = \sum_{i\in I} p_{ai}\tilde{q}_i$. Thus the probability of counting *n* trips during *D* units of time is

$$\Pr(n|\, D, \tilde{x}_a) = \exp(-D\tilde{x}_a)\frac{(D\tilde{x}_a)^n}{n!} \quad .$$

Another traditional assumption is that the traffic count is a gaussian R.V. with mean intensity $\tilde{x}_a$ and known estimated variance $\tilde{v}_a$, leading to the following PDF

$$\Pr(x|\, \tilde{x}_a, \tilde{v}_a) = \frac{1}{\sqrt{2\pi\tilde{v}_a}}\exp(-\tfrac{1}{2}\frac{(x-\tilde{x}_a)^2}{\tilde{v}_a}) \quad .$$

When en-route surveys are available, we can derive the following observation equation

$$\bar{q}_i = \tilde{q}_i + \varepsilon_i$$

in which $\bar{q}_i$ denotes the recovered value and ε_i the observation error, with estimated variance $\tilde{v}_i$. Then under the assumption of normal error we obtain an approximate PDF of

$$\Pr(\bar{q}_i|\, \tilde{q}_i, \tilde{v}_i) = \frac{1}{\sqrt{2\pi\tilde{v}_i}}\exp(-\tfrac{1}{2}\frac{(\bar{q}_i-\tilde{q}_i)^2}{\tilde{v}_i}) \quad .$$

E2b The composition of observations

The overall information consists in a set of individual observations, which should be combined in a consistent way.

A frequent assumption is that the individual observations are independent. Then the likelihood function of the overall information is the product of those of the individual observations, which facilitates the estimation. The most common justification is associated with observations performed in different places or at different times.

When the observation equations derive from en-route O-D surveys, they cannot be assumed to be independent. This mutual dependency can be taken into account by assuming a joint multivariate normal distribution, with estimated covariance matrix $\tilde{C} = [\mathrm{cov}(\bar{q}_i, \bar{q}_j)]_{i,j\in I}$ obtained as indicated in the lesson on Pools and Panels. Then the joint PDF is

$$\Pr(\overline{q}\,|\,\widetilde{q},\widetilde{C}) = \frac{\exp(-\frac{1}{2}(\overline{q}-\widetilde{q})^t\,\widetilde{C}^{-1}(\overline{q}-\widetilde{q}))}{\sqrt{(2\pi)^{|I|}\det\widetilde{C}}}\,.$$

E2c Estimation

The vector of parameters Θ can be estimated by one out of the two standard methods, maximum likelihood (ML) or least squares (LS). Each method amounts to solving a mathematical optimization program with numerous inputs under specific formats.

The estimation results in estimates, i.e. values of the estimator functions $\hat{\Theta}$ which are particular to the sample observation. In the case of ML estimation, asymptotic estimates of accuracy are also obtained and may be used to derive confidence intervals and to test alternative model specifications.

When the specification error η is taken into account, the estimation yields an estimate of its parameters. Then transfer assumptions can be made from observed O-D pairs to unobserved O-D pairs, enabling one to obtain an estimated value and a confidence interval even for an O-D flow which is unobserved, based on observed attributes of its origin and destination zones and of the transport system.

E3 Network assignment of estimated O-D flows

Most often an O-D matrix is not estimated per se, but to serve as input to a network assignment model. In this case it is straightforward to illustrate the results of the estimation by assigning them onto the network, and also to assess the model's relevance by comparing predicted link flows to observed link flows not used in the estimation process: this process is known as model validation.

E3a Assignment of estimated values and variances

Let us assume that there are estimated O-D flow values $\hat{q} = (\hat{q}_i)_{i\in I}$ and also estimated variances $\hat{V} = (\mathrm{var}\,\hat{q}_i)_{i\in I}$. The estimated matrix of flows $\hat{q}$ can be assigned to the associated network, provided a network assignment model be available. This yields link flows

$$\hat{x}_a = \sum_{i\in I} p_{ai}\hat{q}_i\,.$$

Similarly the matrix of flow variances, $\hat{V}$, considered as another O-D trip matrix, can be assigned to the network using link-O-D coefficients p'_{ai}, yielding ling flows of the form

$$\hat{v}_a = \sum_{i\in I} p'_{ai}\,\mathrm{var}\,\hat{q}_i\,.$$

The estimated variance of link flow $\hat{x}_a$ may be developped into

$$\mathrm{var}\,\hat{x}_a = \sum_{i\in I} p_{ai}^2\,\mathrm{var}\,\hat{q}_i + \sum_{i\neq j} p_{ai}p_{aj}\,\mathrm{cov}(\hat{q}_i,\hat{q}_j)$$

$= \sum_{i\in I} p_{ai}^2\,\mathrm{var}\,\hat{q}_i$ by neglecting the covariance of different O-D pairs.

Taking the link-O-D coefficients p'_{ai} equal to p_{ai}^2, we obtain that $\hat{v}_a$ is identical to $\mathrm{var}\,\hat{x}_a$. Then a network plot of the link values $\hat{v}_a$ will reveal the accuracy of the estimated link flows.

E3b Validation again test link flows

The predicting power of the distribution model can be tested by comparing at the link level two different confidence intervals. The first one is a predicted confidence interval $\hat{x}_a \pm t_{\alpha/2}\sqrt{\hat{v}_a}$, based on estimated O-D flows assigned onto the network. The other one is an empiric confidence interval $\hat{x}_a^{\text{emp}} \pm t_{\alpha/2}\,\text{SE}_{\hat{x}a}$ derived from the observation of a random sample of the link flows.

The comparison can be illustrated by a diagram of predicted flows and empiric flows. To each test link are associated two segments on the vertical line with abscissa $\hat{x}_a$, the first segment from point $(\hat{x}_a ; \hat{x}_a - t_{\alpha/2}\sqrt{\hat{v}_a})$ to point $(\hat{x}_a;\hat{x}_a + t_{\alpha/2}\sqrt{\hat{v}_a})$ and the second one from point $(\hat{x}_a;\hat{x}_a^{\text{emp}} - t_{\alpha/2}\,\text{SE}_{\hat{x}a})$ to point $(\hat{x}_a;\hat{x}_a^{\text{emp}} + t_{\alpha/2}\,\text{SE}_{\hat{x}a})$. A nonempty intersection of the two intervals indicates a compatibility between prediction and observation at the 1-α confidence level for that link.

When the observation of the link flow belongs to the estimation data set, the two intervals are expected to share a large intersection. A more interesting case arises when the observation does not belong to the estimation data set: this is referred to as model validation.

E4 About software

E4a General

Specialized software is required to estimate O-D matrices and demand functions. The two basic reasons are the specific formats of the inputs and outputs (information by O-D pair, network link, link and O-D pair) and the large size of the mathematical optimisation program.

Most transportation planning packages include a procedure for maximum entropy inference of O-D trip matrices, based on zonal margins (origin totals, destination totals) and also on link counts, assuming that O-D link proportions are available. Also commonly available are demand functions, provided that the user fixes the values of the parameters.

E4b Instance: the TRIPS package

The TRIPS package includes a module, called MVESTL, dedicated to the estimation of an O-D matrix on the basis of the following information:

- Prior matrix,
- Link counts, origin and destination margins,
- Transit trip matrices between cordon points (eg. data from number plate surveys),
- link proportions for traffic traversing a given node.

To each observation is associated a specific confidence level, fixed by the analyst. The main assumption is that each O-D flow, multiplied by its confidence level, is a Poisson R.V. with parameter

$$T_{od} = a_o b_d t_{od} \prod_k (X_k)^{R_{odk}}$$

in which a_o denotes the origin margin, b_d the destination margin, X_k a quantity such as the number of trips in a class of generalized cost, R_{odk} measures the dependency of the k-th attribute on the O-D pair (*o*, *d*), and t_{od} is either a prior O-D flow or the outcome of a demand function $t_{od} = c_{od}^{\alpha} \exp(-\beta c_{od})$ (where c_{od} denotes the O-D generalized cost and α and β are parameters).

Estimation is performed by maximum likelihood, assuming independent observations. This yields estimates and variability estimates, which can be propagated to estimate O-D flows and their variability. Flow variances can be assigned onto an associated transport network.

F An instance of off-trip survey: the French nationwide Household Travel Survey of 1993

In France, every tenth year or so, a national survey is conducted to reveal the travel practice of the households. The last instance, which was performed in 1993-1994, is called the Transport and Telecommunication Survey since it also includes questions about telecommunication equipment and practice.

We shall present the last French Nationwide Household Travel Survey as an instance of off-trip surveys. It is remarkable owing to the large sample size and to the far-reaching questionnaire. First we shall introduce the questionnaire. Then we shall describe the sample selection and survey administration. Lastly we shall provide various results and applications.

F1 Survey questionnaire

The survey questionnaire is comprised of four parts: household description, person description, vehicle description and trip description. Additional information on the housing environment and the meteorologic conditions of the surveyed days was reported by the surveyors for each household.

F1a Household description

General household information consists in the residence area (municipality), the number of persons in the household, the number of employed persons, the number of persons with driving license, the reference person and his professional or occupation status.

A question ahout the income could eventually be answered by indicating an income class.

The equipment of the household is described on the basis of number of cars, number of bicycles, motorbikes and motorcycles, and also phone equipment.

Additional questions pertain to persons absent for a long time (1% of population, who contribute to at least 5% of the travelled distance in long trips), and to roadway accidents.

F1b Person description

Every person aged six years or more in a sampled household was submitted to an individual questionnaire about socio-economic attributes, transport equipment and mobility habits.

The surveyed socio-economic attributes are the sex, age, level of education, profession.

Individual transport equipment is characterized in terms of driving license (car and motorcycle), driving practice and subscription to public transport networks.

Mobility habits correspond to repeated trips of a given nature, specifically home to work, home to school or learning place, home to nursery.

F1c Vehicle description

A specific questionnaire was associated to each of the following vehicle types: passenger cars and light trucks (up to 3.5 tons), motorcycles, motorbikes and bicycles. Whenever relevant the following items were surveyed:

- Brand and pattern,
- Date of issue and acquisition status (new vs. used),
- Gas type, power of motor,
- Usual night parking place,
- Main user, total travelled distance, travelled distance over the previous year for five different purposes.

F1d Trip description

The characterization of trips was addressed by three questionnaires about respectively the daily mobility, the long distance travel and the car movements.

The questionnaire on daily mobility was addressed to only one person in the household, randomly drawn with equiprobability. This person had to describe the trips done on the day before the surveyor's visit, and also the motorized trips of the previous week-end.

The questionnaire on long distance travel was addressed to only one person in the household, randomly drawn with inequal probabilities: the most mobile person in the household was selectd two times out of three. The respondent had to describe his long distance trips over the previous three months.

The questionnaire on car movements concerned one car in each motorized household, randomly drawn with bias towards the second car. All the trips of the previous week were to be reported.

Whatever the trip questionnaire, each surveyed trip is described by essentially the same attributes, namely:

- The origin and destination places (by municipality),
- The stated distance (eg. from distance counter) and computed euclidean distance,
- The departure and arrival times,
- The transport modes, with up to six different submodes,
- The origin and destination purposes,
- The size of the travelling group and its composition,
- The eventual use of telephone during the trip.

F2 Sample selection and survey administration

F2a Sample size and response rates

A random sample of 20,000 households was drawn from the file of houses resulting from the 1990 population census. About 2,700 sampled houses were either vacant or secondary houses, leading to 17,300 valid households. Almost 82% of these accepted to take part in the survey, yielding a final sample size of 14,200 households.

The collected questionnaires concern 14,200 households, 38,500 persons, 96,000 daily trips, 23,100 long distance sojourns (and 41,800 long distance trips), 19,300 vehicles and 197,000 car movements.

F2b Optimization of sampling plan

The sample was stratified with respect to geographic position (region and type of urban area), period in the year (eight periods from May 1993 to April 1994), and also number of cars in the household, since this information was available from the population census. The sample rate in each stratum favored the multimotorized households.

Inside a sampled household, the inequal random sampling of the long distance respondent favored the most mobile person.

It is estimated that the use of inequal probabilities increased the number of recorder personal trips by about 23%.

F2c Correction of non-response and sample fluctuation

The problem of non-response was addressed by a two-step procedure. Firstly the factors of non-response were analyzed (bias towards nonèresponse in denser areas, single-person households, nonmotorized household, senior reference person), leading to categories of response rates. Secondly the sample was post-stratified: each respondent was given a posterior expansion coefficient equal to the prior expansion coefficient divided by the category response rate.

The sample fluctuation was corrected by adjusting the expansion coefficients to match a number of external criteria, called margins. This process is similar to a multiproportional distribution model based on minimum cross entropy (between posterior and prior weights). The external criteria were the following: professional status, a combined sex and age variable, household size, type of residence area (with respect to center or periphery of urban area), day in the week and survey period.

Additional correction tackled the omission effects and the missing or inconsistent data.

F3 Results and applications

The results of the 1993 French Nationwide Travel Survey have been used in many studies, from the en-route practice of telecocommunication to the time and space localizationof parked cars.

F3a On motorized mobility

The following table indicates some results, either aggregate of disaggregate, with respect to persons (aged of six years or more).

Tab. B. Aggregate vs. Personal motorized travel consumption in France, 1993.

Item	Local trips	Other short trips	Long trips	Total
M trips per week	847	21	14	882
Nb of trips per person and per week	15.9	0.4	0.3	16.6
Average length (km)	9.8	21.4	383	16.1
Km per person and per week	157	8	102	267
Average duration (mn)	19	32	254	23
Average speed (km/h)	31.4	40.1	90.5	42.0
M Voy.km, per week	8 300	450	5 410	14 160

The darkened line corresponds to category trip rates for the three purposes of local trips (i.e. with length less than 100 km, trip ends less than 80 km away from home), other short trips (with length less than 100 km), or long trips.

F3b On modal shares

The following table depicts the travelled distances with respect to transport modes (excluding on foot). It reveals the leadership of passenger car, either as a driver or as a passenger. The contribution of short distance modes (bicycle, local public transport) is modest.

Tab. C. Weekly usage of motorized transport modes by French household.

Transport mode	Local trips	Other short trips	Long trips	Total	%
Bicycle	78	2		80	0.6
Motorbike	45			45	0.3
Motocycle	48	1	14	63	0.4
Car as driver	4 645	499	1 483	6 327	44.7
Car as passenger	1 422	77	964	2 463	17.4
Car out of household	1 110	105	463	1 678	11.9
Bus, coach	451	35	230	716	5.1
Metro, tramway	231	9		240	1.7
Classical train	240	9	366	615	4.3
High Speed Train	0		281	281	2.0
Plane	0		1 472	1 472	10.4
Others	28	8	144	180	1.3
TOTAL	8 299	446	5 415	14 160	100

(In millions of passenger.kilometer per week).

F3c Derivation of O-D flows

38 transport zones were defined, including the 22 French regions and 16 additional zones covering the rest of the world. Under a symmetry assumption, the number of O-D cells is 741, among which 598 are nonempty. The 100 most trafficked O-D pairs contain 82% of all trips.

Regional traffic, i.e. trips internal to a French region, constitutes 21% of trips. As could be expected, large and populated regions are well represented. Also important are the flows between the Paris region and the neighboring zones: 3.8% with region Center, same with region Normandy and even 0.2% with South-East of England.

F3d Comparison to external data sources

Consistency with the 1990 population census was forced by the survey design, from sample selection to margin adjustment.

A comparison to network statistics showed that the surveyed traffic represented 70% of the travelled distance on tolled motorways, 80% on interurban train andn 85% on national airline services.

G The Formulation of Discrete Choice Models

A Discrete Choice Model (DCM) is a model of choice from among several variants. The probability of choosing a given variant is a mathematical function which depends on attributes of the variants and of the decision-maker and which involves parameters. The formulation of a DCM pertains to the specification and economic interpretation of the mathematical function and its arguments, whereas its estimation consists in recovering the parameters on the basis of observations.

This lesson focuses on the formulation of DCM. It is made up of three parts. First we introduce functional relationships linking discrete endogenous variables to exogenous variables, either discrete or continuous. Then we provide the economic interpretation of rational behaviour and utility maximization: this microeconomic foundation, combined to certain aggregation assumptions, leads to several important functional relationships. Lastly we illustrate the formulae and their interpretation by the case of the Prado-Carénage tunnel.

G1 Formulation

G1a Terminology

In a discrete choice model, the significance of the words 'discrete' and 'choice' is as follows. The adjective 'discrete' means that there are a finite (or denumerable) number of positions, all of which are mutually exclusive. In route choice, a position is a path (or route, or strategy), whereas in mode choice a position is a transport mode. The noun 'choice' in itself means a decision process conducted by a decision-maker. In the context of general DCM however, it just refers to a process of assigning a number of entities to a set of positions. The assignment may correspond to an economic process; it may alternalively correspond to a stochastic process.

Formally, a general DCM is a quantitative formula to assign entities to positions, also called variants. The entities can be distinguished from each other up to a limited degree of accuracy: the resulting underdetermination provides a probabilistic foundation to the formula. Of prominent interest are the position market shares, formulated as functions of the attributes of the positions and of the entities.

Let us denote by k a position, c a class of entity, X a vector of attributes (of classes or of positions) and Θ a vector of parameters. The market share formula of position k for class c is as follows, denoting by $F_{k,c}$ a mathematical function:

$$p_{k/c} = F_{k,c}(\Theta, X) .$$

In the economic perspective, from assumptions about attributes X, parameters Θ and function $F_{k,c}$ we derive the market share $p_{k/c}$.

In the statistical perspective, from observations of $p_{k/c}$ and X and under prior assumptions on $F_{k,c}$, the aim is to estimate the unknown values of the parameter vector Θ, and also to predict the accurracy of this estimation.

G1b Generic formulae

The discrete choice of a consumer *c* among variants *k* involves binary endogenous variables $[y_{ck}]_k$ such that $y_{ck} = 1$ if consumer *c* chooses variant *k* or 0 otherwise. Thus $\sum_k y_{ck} = 1$. If we consider a class of *N* consumers $c(i)$, $i \in \{1,2..N\}$, there are *N* vectors of endogenous variables $[y_{c(i),k}]_k$. The relative empiric frequency (market share) of variant *k* is also

$$p_{k/c} = \frac{\sum_{i=1}^{N} y_{c(i)k}}{N}.$$

As $p_{k/c}$ is a continuous variable, it may be analyzed more easily than the individual binary variables y_{ck}.

Formally a DCM is defined as a functional relationship between $p_{k/c}$ and exogenous variables *X* which include class attributes and variant attributes:

$$p_{k/c} = F_{k,c}(\Theta, X), \qquad (1)$$

in which Θ is a vector of parameters and $F_{k,c}$ is a given mathematical function submitted to the normalization constraint $\sum_k F_{k,c}(\Theta, X) = 1$.

Formula (1) indeed is an aggregation formula: the aggregation is conducted on a class of consumers, or more precisely on a class of consumption units.

There may be a second aggregation step when several consumer classes are considered simultaneously: this is stated as

$$p_{k/C} = \sum_{c \in C} p_{k/c} \frac{N_c}{N}, \qquad (2)$$

in which *C* is the set of consumer classes, N_c is the number of consumers in class *c* and $N = \sum_{c \in C} N_c$ is the total number of consumers. When there is a continuous distribution of classes *c*, each one with density of probability *dH(c)*, the second aggregation formula is

$$p_{k/C} = \int_{c \in C} p_{k/c} dH(c). \qquad (2')$$

The distribution of classes may depend on a specific parameter Θ': a latent class model results when the class proportions N_c / N are estimation parameters. In the continuous case, the CDF *H* may also contain estimation parameters.

G1c Discrete choice model under quotient form

In a discrete choice model under quotient form, there exist functions $G_{k,c}$ such that

$$p_{k/c} = F_{k,c}(\Theta, X) = \frac{G_{k,c}(\Theta, X)}{\sum_{k'} G_{k',c}(\Theta, X)}. \qquad (3)$$

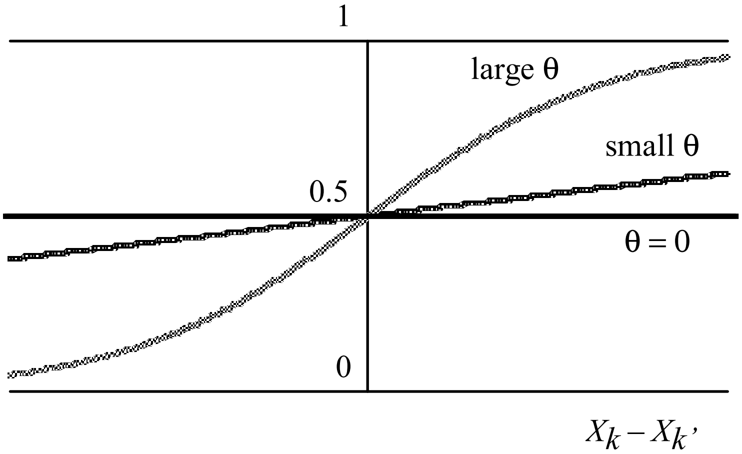
Thus the normalization condition $\sum_k F_{k,c}(\Theta, X) = 1$ is automatically satisfied.

A well-known case is the **linear logit** model, in which $G_{k,c}(\Theta, X) = \exp[\sum_n \theta_n X_n]$ where X_n is the n-th attribute of the c-th consumer class or of the k-th variant and θ_n is the n-th component of the parameter vector Θ.

Let us consider a binary instance of choice among two transport modes $k\in\{1,2\}$ with one mode attribute X_k equal to the average utility (i.e. opposite of generalized cost). Then $G_{k,c}(\Theta, X) = \exp(\theta X_k)$ and $p_{k/c} = \dfrac{\exp(\theta X_k)}{\exp(\theta X_k)+\exp(\theta X_{k'})} = \dfrac{1}{1+\exp(\theta(X_{k'}-X_k))}$.

The value of θ determines the dependency of p_k on the mean utility difference $X_k - X_{k'}$: when it is small the market share increases slowly with respect to $X_k - X_{k'}$, whereas when it is large the market share raises sharply.

Fig. g. Market share of mode *k* as a function of θ and $X_k - X_{k'}$.



A such model is known as a conditional logit model, as opposed to a universal logit in which the function $G_{k,c}$ may also depend on the attributes of other variants.

There exist many related formulations, among which the most widespread are the dogit model which includes saturation effects and the Box-Cox or Box-Tukey models which relax the linear assumption on parameters.

A typical **dogit model** is as follows, letting η_{kc} denote the minimal market share of variant *k* for consumer class *c* (of course $\eta_{kc}\geq 0$ and $\sum_k \eta_{kc} \leq 1$):

$$p_{k/c} = \eta_{kc} + (1-\textstyle\sum_{k'\in K}\eta_{k'c})\dfrac{G_{kc}}{\sum_{k'\in K} G_{k'c}}. \quad (4)$$

In a **Box-Cox** or **Box-Tukey** logit model, the function $G_{kc} = \sum_n \theta_n B^{[n]}(X_{kn})$ is a linear combination of terms $B^{[n]}$ which may be non linear. Precisely,

- a Box-Cox term with parameter λ is $B^{[n]}(x) = \frac{x^\lambda - 1}{\lambda}$ if $\lambda > 0$ or $B^{[n]}(x) = \ln x$ if $\lambda = 0$.
- a Box-Tukey term with parameters λ and μ is $\frac{(x+\mu)^\lambda - 1}{\lambda}$ if $\lambda > 0$ or $\ln(x+\mu)$ if $\lambda = 0$.

G1d Discrete choice model under distribution form

A DCM under distribution form is based on real random variables Y_{kc} associated to each variant *k* and each consumer class *c*, as inputs to the following market share formula

$$p_{k/c} = \Pr(Y_{kc} \geq Y_{k'c}\ \forall k' \in K)\,. \tag{5}$$

This includes the **binary price-time model**, in which consumer *c* with value-of-time (VoT) v_c associates to mode *k* the utility function $Y_{kc} = -P_k - v_c T_k$ wherein P_k is the price and T_k the travel time. In this case the random variable $Y_k = -P_k - vT_k$ derives its randomness from that of *v*, provided that *c* is selected at random. Let *H* denote the CDF of the VoT *v*:

$$p_k = \Pr(Y_k \geq Y_{k'}) = \Pr((T_k - T_{k'})v \leq P_{k'} - P_k)\,. \tag{6}$$

Let us assume that $T_k < T_{k'}$, hence *k* is the faster mode. Letting $v_{\text{cut}} = \frac{P_k - P_{k'}}{T_{k'} - T_k}$ denote a cut-off, supply-related value-of-time, we obtain that

$$p_k = 1 - H(v_{\text{cut}})\,. \tag{7}$$

This model is known as a **varying parameter** model, since the parameter *v* varies across the population of consumers.

Another instance is the **linear probit** model, in which Y_{kc} decomposes into a linear combination of attributes $\theta X = \sum_n \theta_n X_{kn} + \sum_m \theta_m X_{cm}$ and a random variable ε_{kc} such that the random vector $(\varepsilon_{kc})_k$ is multivariate normal. In the binary case, the difference $\Delta\varepsilon = \varepsilon_{k'c} - \varepsilon_{kc}$ is also gaussian with mean $\Delta\bar{\varepsilon}$ and standard deviation $\sigma_{\Delta\varepsilon}$, hence

$$p_k = \Pr(Y_k - Y_{k'} \geq 0) = \Pr(\Delta\varepsilon \leq -\theta(X_{k'} - X_k)) = \Phi\left(\frac{-\theta(X_{k'} - X_k) - \Delta\bar{\varepsilon}}{\sigma_{\Delta\varepsilon}}\right) \tag{8}$$

where Φ is the CDF of a reduced gaussian distribution (with null mean and unit variance).

G1e Compound distribution DCM

A compound distribution DCM arises when the random variables Y_{kc} are made up of several elementary random variables. For instance

$$Y_k(v,\omega) = -P_k - vT_k + \varepsilon_{k/v}(\omega) \tag{9}$$

where the couple (v,ω) denotes a compound random event, *v* is a random variable and $\varepsilon_{k/v}$ is a random variable conditional on *v*.

There may be no closed-form formula for p_k in this case. In the binary case, assuming that *v* has CDF *H* and $\Delta\varepsilon = \varepsilon_{k'c} - \varepsilon_{kc}$ has conditional CDF Z_v, as $Y_k \geq Y_{k'}$ is equivalent to $\Delta\varepsilon \leq -\Delta P - v\Delta T$ it holds that

$$p_k = \int_v Z_v(-\Delta P - v\Delta T)\, dH(v)\,. \tag{10}$$

When *v* is uniformly distributed on [*A*,*B*] (i.e. $dH(v) = \frac{1}{B-A}dv$ on [*A*,*B*] and 0 elsewhere) and Z_v admits a primitive function $\tilde{Z}_v$ under closed-form, we obtain a closed-form formula

$$p_k = \frac{1}{B-A}\int_A^B Z_v(-\Delta P - v\Delta T)\, dv = \frac{\tilde{Z}_v(-\Delta P - B\Delta T) - \tilde{Z}_v(-\Delta P - A\Delta T)}{(B-A)\Delta T}\,. \tag{11}$$

G1f Hierarchical DCM

In a multinomial DCM, the assignment of a consumer to a variant is a single-step process. When there are numerous variants, a better solution may be to identify subsets of variants and to model the assignment as a multi-step process, eg. first selection of subset and then selection of variant within subset.

A well-known instance is the modal competition between the car, a first bus service called the blue bus and a second one called the red bus. To the urban trip-maker the difference between the car and any bus service is far larger than that between the red bus and the blue bus. This may be accomodated by identifying a subset of "Bus services".

Let us denote by *S* the set of subsets, and by *s* a given subset of variants *k*. A two-step DCM is

$$p_{k/c} = p_{s/c}\, p_{k/sc}, \tag{12}$$

in which $p_{k/sc}$ is the conditional probability of choosing variant *k* out of subset *s*.

A **nested logit** model arises when both $p_{s/c}$ and $p_{k/sc}$ have the logit form.

G1g On exogenous variables

The value of a continuous real variable indicates a position on the real axis. The distance between two positions has a physical or economic significance, let us say a geometric significance.

In the case of a discrete variable or qualitative nature, the relative positions of the modalities have no geometric significance. However a discrete variable D_n may be included in a quantitative function $F_{k,c}$ by way of "dummy variables" $z_{n,i}$ associated with each modality *i* of D_n by assuming that $z_{n,i} = 1$ is modality *i* is satisfied or $z_{n,i} = 0$ otherwise. Thus it always holds that $\sum_{i \in I_n} z_{n,i} = 1$. Then real coefficients $\theta_{n,i}$ can be associated to the dummy variables $z_{n,i}$ and the product $\theta_{n,i} z_{n,i}$ may contribute to a function $F_{k,c}$.

G2 Economic interpretation

A DCM has an economic content if it is possible to provide an economic interpretation to the entities, the positions and the assignment process.

G2a Basic microeconomic interpretation

The entities are consumers of a product, of which the positions are alternative, competing variants. The assignment process is an economic choice based on the individual consumer's preferences: each consumer is a preference-optimizing decision-maker. Thus the behavioural model is based on the **axiom of eonomic rationality**.

To the axiom is added the **assumption of transitive preferences**, which states that the preferences of a decision-maker can be expressed by a utility function, i.e. an overall rating which the decision-maker associates to each available variant.

G2b Random utility theory

In the theory of random consumer utility, it is assumed that consumers choose from among variants in a joint economic and random way. Precisely, each consumer *c* associates to each variant *k* a random utility function $U_{k,c}$ and he chooses variant *k* on every occasion such that $U_{k,c} \geq U_{k',c}$ for every variant *k'*.

Thus the market share of variant *k* to consumer *c* is $\Pr(U_{k,c} \geq U_{k',c}\ \forall k')$, which is individual with respect to the consumer but aggregate with respect to the random occasions.

Function $U_{k,c}$ may include attributes of the variants and of the consumer. Most often the analyst prefers not to include attributes of other variants. A conventional, simple linear utility function is as follows

$$U_{k,c} = \sum_{n \in I} \theta_n X_n(k) + \sum_{m \in J} \theta_m X_m(c) + \varepsilon_{k,c}(\omega)$$

in which *I* and *J* are disjoint subsets of components of Θ, attributes $X_n(k)$ relate to variant *k*, attributes $X_m(c)$ relate to consumer *c*, ω denotes an elementary random event and $\varepsilon_{k,c}$ is a random variable.

This linear formula separates the respective influence of variant, consumer and randomness in an additive way. We can interpret its deterministic part $\overline{U}_{k,c} = \mathrm{E}_{\omega}[U_{k,c}] = \sum_{n \in I} \theta_n X_n(k) + \sum_{m \in J} \theta_m X_m(c) + \overline{\varepsilon}_{k,c}$ as a generalized cost, up to a minus sign. Provided that the time T_k and the price P_k are included in the first part of $U_{k,c}$, the ratio θ_T / θ_P of their coefficients may be interpreted as the value-of-time of consumer *c*. However in most cases the estimation is based on a segmentation of all consumers into classes and the parameters θ_T and θ_P relate to either the whole set of consumers, or a given class. Therefore the ratio θ_T / θ_P is above all an average (mean or median) value-of-time.

A random variable $\varepsilon_{k,c}$ is comprised of its mean $\overline{\varepsilon}_{k,c}$ and a random residual $\varepsilon'_{k,c}(\omega) = \varepsilon_{k,c}(\omega) - \overline{\varepsilon}_{k,c}$. The mean is called the modal constant of variant *k* and it is usually included in the deterministic part of the utility function by way of a dummy variable: it reflects the aggregate effect of all unobserved factors. The random residual can be interpreted as the analyst's uncertainty on explicit attributes *X*, or as the uncertainty in the consumer's evaluation of the variant. In the case of repeated choice by a given consumer, the latter part may be due to temporal variations (eg. of the variant attributes).

G2c Derivation of logit and probit model

The logit and probit models can be obtained as random utility models under specific assumtions.

In the **linear logit** case, a linear utility function is assumed in which the residual variables $\varepsilon'_{k,c}$ are independent and have identical Gumbel distributions with variance $\pi^2/(6\theta^2)$. Thus their common cumulative distribution function is $G(x) = \exp(-\exp(-\theta x + \gamma))$ where $\gamma \cong 0.577$ is Euler's constant. Then, denoting by $\overline{U}_{k,c}$ the deterministic part of the utility function, it holds that

$$\Pr(U_{k,c} \geq U_{k',c}\ \forall k') = \frac{\exp(\theta \overline{U}_{k,c})}{\sum_{k'} \exp(\theta \overline{U}_{k',c})}.$$

In the **linear probit** case, a linear utility function is assumed in which the residual variables $\varepsilon'_{k,c}$ are multivariate normal with matrix of covariance M. This enables one to consider joint variations of the $\varepsilon'_{k,c}$. In the binary case, assuming independent residuals $\varepsilon'_{1,c}$ and $\varepsilon'_{2,c}$ with variance σ_1^2 and σ_2^2 respectively, the first variant is chosen with probability $\Pr(U_{1,c} \geq U_{2,c})$ $= \Pr(V_1 + \varepsilon'_1 \geq V_2 + \varepsilon'_2)$ $= \Pr(\varepsilon'_2 - \varepsilon'_1 \leq V_1 - V_2)$ $= \Phi((V_1 - V_2)/\sqrt{\sigma_1^2 + \sigma_2^2})$ in which Φ is the CDF of a reduced gaussian random variable and $\sigma_1^2 + \sigma_2^2$ is the variance of the gaussian variable $\varepsilon'_2 - \varepsilon'_1$.

G2d Derivation of varying parameter DCM

Varying parameter DCM are derived from random utility theory under the additional assumption of aggregation with respect to unobserved attributes of the consumers.

G2e Derivation of nested logit

Recall that in a nested logit mdel, the variants *k* are grouped into subsets *s*. The lower-level conditional choice within a subset *s* involves a random utility function $U_k(\omega)$ = $\overline{U}_k + \varepsilon_{k/s}(\omega)$ with a deterministic part $\overline{U}_k$ and a random term $\varepsilon_{k/s}(\omega)$ (which is conditional on the choice of subset *s*).

The upper-level choice among subsets involves random utility functions $U_s(\omega) = \overline{U}_s + \varepsilon_s(\omega)$.

For the sake of economic consistency, the deterministic part $\overline{U}_s$ must decompose into $\max_{k \in s} U_k(\omega)$ and a function of attributes of the decision-maker and of the subset only. Assuming independent, identically distributed Gumbel variables $\varepsilon_{k/s}$ with variance $\sigma_s^2 = \pi^2/(6\theta^2)$, it holds that

$$\max_{k \in s} U_k(\omega) = \tfrac{1}{\theta} \ln \textstyle\sum_{k \in s} \exp(\theta \overline{U}_k), \tag{13}$$

which is called the log-sum formula.

Economic consistency also requires σ_s to be inferior to the standard deviation of the random variable ε_s.

G2f Economic outreach

The economic outreach of a discrete choice model depends on the explicit and accurate description of economic choices. The description is as much explicit as there are explicit dimensions of analysis, eg. O-D pair, trip purpose, mobile type and VoT. Its accuracy depends on whether the dimensions of analysis have straightforward economic significance (eg. VoT more meaningful than trip purpose) and whether the economic parameters correspond to the preferences of an individual decision-maker (particular choice better than the mix of several choices).

G3 Case study of the Prado-Carénage tunnel

The Prado-Carénage tunnel in Marseilles is a toll, underground, car-only motorway in the French city of Marseilles. Several binary route choice models were designed to analyze the competition between the toll routes using the tunnel and alternative, toll-free routes. We shall first develop the most general form, and then describe several simplified models.

G3a The economic model and its interpretation

The behavioural model is based on the axiom of economic rationality: this states that the decision-maker chooses between the available options on the basis of his or her personal preferences. In the context of route choice on road network the decision-makers are drivers who decide which path to take: they are assumed to select the most advantageous path, i.e. the quickest or the least expensive path or, what is more likely, a path which represents a compromise between the shortest time and the lowest cost.

The hypothesis of transitive preferences states that the preferences of a decision-maker can be expressed by a utility function, i.e. an overall rating which the decision-maker assigns to each available option. This leads us to represent the utility of the tolled route for the driver i by $\mathrm{U}_{\mathrm{toll}}(i)$ and that of the toll-free route by $\mathrm{U}_{\mathrm{free}}(i)$. The hypothesis of rationality is expressed as follows: if $\mathrm{U}_{\mathrm{toll}}(i) \geq \mathrm{U}_{\mathrm{free}}(i)$ the driver will choose the tolled route but if $\mathrm{U}_{\mathrm{toll}}(i) < \mathrm{U}_{\mathrm{free}}(i)$ the driver will choose the toll-free route. Everything therefore depends on how the utility difference function $\Delta\mathrm{U}(i) = \mathrm{U}_{\mathrm{free}}(i) - \mathrm{U}_{\mathrm{toll}}(i)$ is specified.

We shall represent the difference in utility between the toll-free route and the tolled route by a model of the following form, in which v_i is the VoT for driver i (for this particular trip), θ_D is a value-of-distance, $\mathrm{P}_{\mathrm{free}}$ and $\mathrm{P}_{\mathrm{toll}}$ are the prices of the two routes, $\mathrm{T}_{\mathrm{free}}$ and $\mathrm{T}_{\mathrm{toll}}$ are their travel times, D_{free} and D_{toll} their distances and $\Delta\varepsilon = \varepsilon_{\mathrm{free}} - \varepsilon_{\mathrm{toll}}$ is the difference between the random errors for each route:

$$\Delta\,\mathrm{U}(i) = -(\mathrm{P}_{\mathrm{free}} - \mathrm{P}_{\mathrm{toll}}) - v_i(\mathrm{T}_{\mathrm{free}} - \mathrm{T}_{\mathrm{toll}}) - \theta_D(D_{\mathrm{free}} - D_{\mathrm{toll}}) + \Delta\varepsilon\,. \qquad (14)$$

Formula (14) takes account of the diverse trade-offs between price and journey time as the VoT v_i depends on the consumer. The random variable $\Delta\varepsilon$ encompasses the additional elements which are not taken into account in an explicit way.

G3b Simplified models

The following table describes simplified models of the base model.

A single VoT is denoted by a parameter $\bar{v}$. A log-normal VoT v has two parameters, namely the mean μ and standard deviation σ_v of the random variable $\ln v$ which is gaussian. A log-logistic VoT v has two parameters a, b such that $H(v) = 1/(1+bx^{-a})$, yielding a mean of $b^{1/a}\,\dfrac{\pi/a}{\sin(\pi/a)}$ and a standard deviation of $b^{1/a}\,\dfrac{\pi/a}{\sin(\pi/a)}(\frac{a}{\pi}\tan(\frac{\pi}{a}) - 1)^{1/2}$.

Model	Parameters	Constraints
Logit with single VoT	$\Delta\bar{\varepsilon}$, $\sigma_{\Delta\varepsilon}$, $\bar{v}$, (θ)	
Probit with single VoT	$\Delta\bar{\varepsilon}$, $\sigma_{\Delta\varepsilon}$, $\bar{v}$, (θ)	
Log-normal VoT, $\Delta\varepsilon = 0$	μ, σ_v, (θ)	$\sigma_v \geq 0$
Log-logistic VoT, $\Delta\varepsilon = 0$	a, b, (θ)	$a \geq 0$, $b \geq 0$
Logit with log-normal VoT	$\Delta\bar{\varepsilon}$, $\sigma_{\Delta\varepsilon}$, μ, σ_v, (θ)	$\sigma_{\Delta\varepsilon} \geq 0$, $\sigma_v \geq 0$
Logit with log-logistic VoT	$\Delta\bar{\varepsilon}$, $\sigma_{\Delta\varepsilon}$, a, b, (θ)	$\sigma_{\Delta\varepsilon} \geq 0$, $a \geq 0$, $b \geq 0$
Logit with uniform VoT	$\Delta\bar{\varepsilon}$, $\sigma_{\Delta\varepsilon}$, A, B, (θ)	$\sigma_{\Delta\varepsilon} \geq 0$, $B - A \geq 0$
Probit with log-normal VoT	$\Delta\bar{\varepsilon}$, $\sigma_{\Delta\varepsilon}$, μ, σ_v, (θ)	$\sigma_{\Delta\varepsilon} \geq 0$, $\sigma_v \geq 0$
Probit with log-logistic VoT	$\Delta\bar{\varepsilon}$, $\sigma_{\Delta\varepsilon}$, a, b, (θ)	$\sigma_{\Delta\varepsilon} \geq 0$, $a \geq 0$, $b \geq 0$
Probit with uniform VoT	$\Delta\bar{\varepsilon}$, $\sigma_{\Delta\varepsilon}$, A, B, (θ)	$\sigma_{\Delta\varepsilon} \geq 0$, $B - A \geq 0$

H Integration of RP and SP data

The specific advantages of SP surveys are balanced by uncertain reliability, since they result in hypothetical response. To overcome this drawback, SP data may be combined to related RP data in a number of ways.

This lesson describes several ways to combine SP and RP data. It is comprise of three parts. First the specific advantages and disadvantages of RP and SP are emphasized. Then we introduce three basic combinations, namely reweighting, constrained estimation and pooling of results. Lastly we focus on the joint estimation of RP and SP choice models, which is the most elaborate combination.

H1 An assessment of Revealed and Stated Preferences

H1a The interpretation of RP observations

In transport demand analysis, RP observations consist in measurements of traffic variables (flows, travel times), eventually disaggregated according to O-D pair, demand class, time period etc. Such measurements characterize the past or current state of the transport system.

Application to prediction requires some modeling assumptions. In the economic theory of choice, the basic interpretation of Revealed Preferences is the following one: that the observed behaviour of an individual decision-maker is the outcome of a rational, cost-minimizing choice between the chosen option and alternative options. The analyst describes the chosen and alternative options and he assumes that the decision-maker is informed on all of them. This amounts to supplementing the observation of individual outcome with information on alternative options, which is synthetic with respect to the individual decision-maker even if it derives from observations.

The main advantages of RP lies in the observation of all basic attributes. However the following deficiencies may arise:

1. inaccurate measurement of the basic attributes,
2. the subjective perception of attributes by an individual decision-maker may differ from the measurable values,
3. discrepancy between the choice set defined by the analyst and that experienced by the individual decision-maker,
4. some attributes may be highly correlated (eg. travel time and travel distance),
5. some attributes may lack variance in the sample.

The last two deficiencies may threaten the estimation of a choice model.

Another trouble pertains to the introduction of a novel option in a future scenario. If similar options already exist and are part of other individuals' choice sets, then the estimated behavioural parameters may apply to the novel option. However, when the novel option does not exist and is actually unobserved, RP do not apply to it, unless interpreted by the analyst on the basis of analogy and subjective plausibility.

H1b Evaluation of SP data

SP data consist in individual choices (or rating, or ranking) from among a totally synthetic choice set. This results in specific features:

1. totally controlled choice set,
2. availability of sophisticated attributes (eg. variability in travel time, relative frequency of delay),
3. totally controlled attribute levels, allowing for artificial variations to overcome attribute correlation or lack of variance,
4. no measurement error.

However the advantages of total control are balanced by the following sources of bias:

1. information bias since the synthetic choice set may not correspond to that encountered or identified in an observable situation,
2. implementation bias because some important options' attributes may be omitted or described incorrectly in the SP survey as compared to actual implementation (eg. availability of space for luggage in airport bus shuttle),
3. perception bias about attribute levels as interpreted by the decision-maker or as observable on-field.

To limit the potential biases, the common strategy is to base the controlled choice set on the grounds of a real choice situation recently experienced by the individual respondent.

Conversely, as regards the implementation of a novel option studied by SP, the information bias may be reduced by giving the relevant information to all potential customers, whereas the implementation bias may be reduced by adapting the option to its SP specifications.

H2 Basic combinations

Let us now discuss three basic ways to combine information from different sources, particularly RP and SP surveys.

H2a Sample weighting

Any statistical sample may be weighted to match a number of aggregate, observed data, such as classifications with respect to socio-economic attributes derived from a population census.

This concerns the sample selection as well as sample reweighting, i.e. the adjustment of prior individual weights to match one or several simple classifications (eg. with respect to sex, age group, professional status, number of persons in household), yielding posterior expansion coefficient.

The basic method for marginal reweighting is to adjust the observation weights to match the marginal groupings, subject to consistency constraints (positiveness of posterior weights) and in a minimal manner. This is formulated as minimizing a mathematical function of distance between posterior and prior weights, subject to consistency constraints and marginal grouping constraints. A well-known instance in transportation is the minimisation of the cross entropy of an O-D matrix subject to marginal constraints, in which the mathematical function is the cross entropy function.

H2b Constrained estimation of a statistical model

The parameters of a statistical model (RP or SP or else) may be estimated subject to not only formal constraints (eg. positiveness of a standard deviation, normalization of a set of probabilities) but also to empiric constraints, for instance observed aggregate market shares.

The addition of external information will decrease the mean square error of the estimators, maybe at the expense of some bias: the overall result is to increase the empiric outreach of the model. However external empiric constraints often complicate the mathematical program of estimation, because they typically involve more terms than formal constraints and in a more sophisticated fashion.

H2c Pooling the results of several models

Let us consider several models m involving parameters Θ_m with a common interpretation Θ and estimated on independent sets of observations (revealed or stated). As the observations are independent, so are the estimators $\hat{\Theta}_m$ of the common parameter Θ.

The independent R.V.s $\hat{\Theta}_m$ may be pooled to yield an overall estimator $\hat{\Theta}$ based on all observations, with sample size n equal to the sum of sample sizes n_m and also more consistency than the separate estimators.

Assuming large separate sample sizes n_m, the estimators $\hat{\Theta}_m$ are approximately normal R.V.s with same mean μ and particular covariance matrix C_m, estimated by the covariance matrix $\hat{C}_m$ evaluated on the m-th sample.

A bayesian pooled estimator is $\hat{\Theta} = [\sum_m \hat{C}_m^{-1}]^{-1}[\sum_m \hat{C}_m^{-1}\hat{\Theta}_m]$ with estimated covariance matrix $[\sum_m \hat{C}_m^{-1}]^{-1}$.

However the pooling of estimators is restricted to models with a common vector of parameters.

H3 Joint estimation of RP and SP models

A more sophisticated combination consists in estimating a choice model with a twofold set of observations, RP and SP.

H3a Model specification

Let us consider a model of individual choice based on utility functions according to option and decision-maker. Options are indexed by k, whereas individual decision-makers are indexed by i. The attributes of decision-maker i and option k are denoted by three vectors: $\mathbf{x}_{ki}$ for attributes common to RP and SP, $\mathbf{y}_{ki}$ specific to RP and $\mathbf{z}_{ki}$ specific to SP. The level of the attributes $\mathbf{x}_{ki}$ may vary between revealed and stated observations, which is indicated by a superscript R or S.

The model also includes trade-off parameters (α, β, γ) associated to attributes $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ respectively. The random part of the utility function of option k to individual i is denoted by ε_{ki}^{R} or ε_{ki}^{S}.

The RP model is formulated as $U_{ki}^{\mathrm{R}} = \alpha.\mathbf{x}_{ki}^{\mathrm{R}} + \beta.\mathbf{y}_{ki} + \varepsilon_{ki}^{\mathrm{R}}$ for all *k* and *i*, whereas the SP model is formulated as $U_{ki}^{\mathrm{S}} = \alpha.\mathbf{x}_{ki}^{\mathrm{S}} + \gamma.\mathbf{z}_{ki} + \varepsilon_{ki}^{\mathrm{S}}$.

In both models, the terms ε account for unobserved attributes are more generally noise. Their statistical specification implies the important assumptions of:

- random effects of unobserved attributes, resulting in R.V.,
- independence of the variables $\varepsilon_{ki}^{\mathrm{M}}$ for all *k*, *i* and M∈ {R, S},
- identical Gumbel distribution of variables $\varepsilon_{ki}^{\mathrm{R}}$ for all *k*, *i*,
- identical Gumbel distribution of variables $\varepsilon_{ki}^{\mathrm{S}}$ for all *k*, *i*.

We may distinguish the variance of an from that of an $\varepsilon_{ki}^{\mathrm{S}}$ by means of a scale parameter λ > 0 such that $\mathrm{var}\,\varepsilon_{ki}^{\mathrm{R}} = \lambda^2\,\mathrm{var}\,\varepsilon_{ki}^{\mathrm{S}}$.

The vector of parameters is Θ = (α, β, γ, λ). The options' market shares are modelled as follows:

$$\Pr(k \mid i, \mathrm{R}) = \frac{\exp(\alpha.\mathbf{x}_{ki}^{\mathrm{R}} + \beta.\mathbf{y}_{ki})}{\sum_{k' \in \mathrm{R}} \exp(\alpha.\mathbf{x}_{k'i}^{\mathrm{R}} + \beta.\mathbf{y}_{k'i})}$$

$$\Pr(k \mid i, \mathrm{S}) = \frac{\exp(\lambda\alpha.\mathbf{x}_{ki}^{\mathrm{S}} + \lambda\beta.\mathbf{y}_{ki})}{\sum_{k' \in \mathrm{S}} \exp(\lambda\alpha.\mathbf{x}_{k'i}^{\mathrm{S}} + \lambda\beta.\mathbf{y}_{k'i})}.$$

H3b Estimation procedure

The choice model can be estimated by the maximum likelihood method, with a likelihood function equal to the product of the RP and SP likelihoods.

Assuming independent observations, the resulting estimator is consistent and efficient, whereas if the observations are not independent then the estimator is still consistent byt the estimated covariance matrix of estimators is incorrect.

As the presence of λ makes the utility function nonlinear in Θ, maximization of the compound log likelihood requires a special procedure. A program for estimation of hierarchical logit is appropriate, so is the following sequential method:

1. Maximize the SP likelihood to obtain $\overline{\lambda\alpha}^{\mathrm{ML}}$ and $\overline{\lambda\gamma}^{\mathrm{ML}}$.
2. Maximize the RP likelihood with estimated utility $U_{ki}^{\mathrm{R}} = \frac{1}{\lambda}\overline{\lambda\alpha}^{\mathrm{ML}}.\mathbf{x}_{ki}^{\mathrm{R}} + \beta.\mathbf{y}_{ki} + \varepsilon_{ki}^{\mathrm{R}}$, yielding $\hat{\lambda}^{\mathrm{ML}}$ and $\hat{\beta}^{\mathrm{ML}}$. Let $\hat{\alpha}^{\mathrm{ML}} = \overline{\lambda\alpha}^{\mathrm{ML}} / \hat{\lambda}^{\mathrm{ML}}$ and $\hat{\gamma}^{\mathrm{ML}} = \overline{\lambda\gamma}^{\mathrm{ML}} / \hat{\lambda}^{\mathrm{ML}}$.
3. Pool the RP and SP observations after dividing $\mathbf{x}_{ki}^{\mathrm{S}}$ and $\mathbf{z}_{ki}$ by $\hat{\lambda}^{\mathrm{ML}}$, and estimate $\hat{\hat{\alpha}}$, $\hat{\hat{\beta}}$ and $\hat{\hat{\gamma}}$.

H3c Prediction with the joint model

Prediction may be based on the both the RP and SP models. Precisely, the error term comes from the RP model, whereas the attributes are common or specific and parameters derive from the joint estimation.

Then the utility function of option *k* to individual *i* is

$$\hat{U}_{ki}^{\mathrm{R}} = \hat{\alpha}.\mathbf{x}_{ki} + \hat{\beta}.\mathbf{y}_{ki} + \hat{\gamma}.\mathbf{z}_{ki} + \varepsilon_{ki}^{\mathrm{R}}.$$

H3d Discussion

Several points in the abovementioned model for joint estimation and prediction deserve emphasis and discussion.

Does the random term in the revealed (resp. stated) utility function include the contribution and variability of the specific stated attributes $\mathbf{z}$ (resp. specific revealed attributes $\mathbf{y}$)? If positive then in the prediction model the variance of the random error shoud decrease on including specific stated attributes.

Il the utility functions contain modal constants, should these be considered as common parameters α, or as specific parameters appearing in both β and γ? The former case implies that the average contribution of the specific revealed attributes is null, and so is that of the specific stated attributes. The latter case casts much doubt on the prediction method, since the inclusion of some specific stated attributes in the revealed utility function may not be consistent with the revealed modal constant.

Based on this argument, we recommend to estimate jointly a RP and a SP model only when all atttributes in the RP model also belong to the SP model, and to base predictions on the SP model only. Then consistency may be checked by the estimate of the scale parameter λ, presumably larger than one if the RP model is imbedded into the SP model.

I References

I1 In English

Clavering J. and Kirby H. (199x) Data accuracy as an aid to model development – towards a manual of guidance. *Proceedings of the European Transport Forum*, pp. 163-174. PTRC, London.

Greene D.L., Hu P.S. and Roberts G.F. (1984) Analysis of Geographical and Temporal Variation in Vehicle Classification Count Statistics. *Transportation Research Record*, Vol. 987, pp. 21-28.

Hall F.L., V.F. Hurdle and J.H. Banks (1992). *A Synthesis of Recent Work on the Nature of Speed-Flow and Flow-Occupancy Relationships on Freeways*. Paper presented at the 71th Annual Meeting of the TRB (reference 920156). Transportation Research Board, Washington, 25 pages.

Highway Capacity Manual (1985). *TRB Special Report 209*, Transportation Research Board, Washington.

Leurent, F. (1996). An analysis of modeling error, with application to a traffic assignment model with continuously distributed values of time, *Proceedings of the 24th European Transport Forum: Transportation Planning Methods*, **D1**, PTRC, London.

Leurent, F. (1998). *Sensitivity and error analysis of the dual criteria traffic assignment model*. Transportation Research B, **32**/3, 189-204.

MacCarley C.A., Hockaday S.L.M., Need D. and Taff S. (1992) Evaluation of Video Image Processing Systems for Traffic Detection. *Transportation Research Record*, Vol. 1360, pp. 46-49.

Morikawa, T., Ben-Akiva M. and Yamada K. (1991) *Forecasting intercity rail ridership using revealed preference and stated preference data*. Paper presented at the 70th Annual Meeting of the Transportation Research Board, Washington DC.

Ollmann L.T., Howe S.M., Kloeber K.W. and Cohen G.S. (1978) Marginal weighting of transportation survey data. *Transportation Research Record*, Vol. 677, pp. 73-76.

Siaurusaitis V.J. and Saben L.J. (1998) Case Study in the Application of Adjusted Census Data for Planning Projects. *Transportation Research Record*, Vol. 1617, pp. 56-63.

Stokes R.W. and Chira-Chavala T. (1989) Design and Implementation of Intercity Origin-Destination Surveys. *Transportation Research Record*, Vol. 1236, pp. 23-33.

Taylor, M.A.P., & Young, W. (1988). *Traffic Analysis: new technology and new solutions*. Hargreen, Victoria, Australie.

TRB (1983). *Forecasting Inputs to Transportation Planning*. NCHRP Report **266**. TRB, Washington DC.

Walker W.T. (1989) Method to Synthesize a Full Matrix of Interdistrict Highway Travel Times from Census Journey-to-Work Data. *Transportation Research Record*, Vol. 1236, pp. 50-58.

Watling, D.P., & Maher, M.J. (1992). A statistical procedure for estimating a mean origin-destination matrix from a partial registration plate survey. *Transportation Research B*, **26**/3, 171-193.

Wyley, W.J., Hardcastle,D.J. , & Fearon, E.J. (1981). Traffic data variability and its incorporation into model validation procedures. *Traffic Engineering and Control (August/September)* , 473-477.

I2 In French

Armoogum, J. et Madre, J.L. (1998) *Redressement de l'enquête transports pour l'estimation de matrices origine-destination*. Rapport de recherche INRETS n°223. INRETS, Arcueil.

Calzada, C., Marchal, P., Morellet, O. et Soleyret, D. (1997). Evolution, du trafic français de voyageurs à longue distance. Tendances passées et orientations futures. *RTS* 56, pp. 43-60.

Cohen S. (1990). *Ingénierie du trafic routier*. Presses de l'Ecole Nationale des Ponts et Chaussées, Paris.

Hivert, L. (1993) Achat de voiture, diésélisation et kilométrages des ménages. *Les Cahiers Scientifiques du Transport*, 28, pp. 81-98.

Hivert, L. (1998) *Le parc automobile des ménages, étude en fin d'année 1996 à partir de la source Parc Auto SOFRES*. Rapport de convention INRETS pour l'ADEME.

Leurent F. (1993). *Proposition d'indicateurs macroéconomiques des conditions de circulation sur un réseau routier, le cas du boulevard périphérique parisien en 1992*. INRETS Research Report #167. INRETS, Arcueil (50 pages + appendices).

Madre, J.L. (1997) L'enquête transports, *RTS* 56, 6-8.

Madre, J.L. et Maffre, J. (1997) La mobilité des résidants français, Panorama général et évolution. *RTS* 56, pp. 9-26.

Saporta, G. (1990). *Probabilités, Analyse des données et Statistique*. Technip, Paris.

Wonnacott, T.H., & Wonnacott, R.J. (1991). *Statistique*. Economica, Paris.

RTS (1997). Les volumes 56 et 57 de la Revue Transport Sécurité traitent de l'enquête nationale Transport et Télécommunication de 1993-94. Lire notamment l'éditorial de J.P. Orfeuil et les articles déjà cités.